UNDERSTANDING TIDES

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by

Steacy Dopp Hicks
Physical Oceanographer

Center for Operational Oceanographic Products and Services
Michael Szabados, Director

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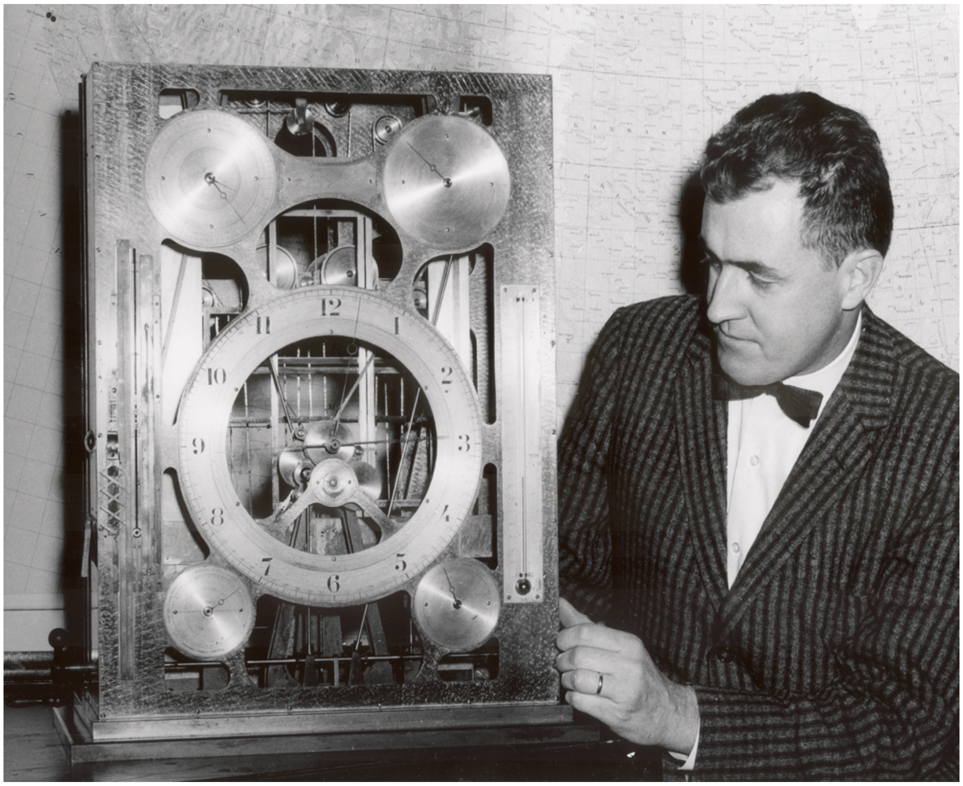

The author, Steacy D. Hicks viewing the William Ferrel tide-predicting machine, the first tide predicting machine used in the United States (1885-1914). Photograph taken in early 1960s.

ABOUT THE AUTHOR

Steacy Dopp Hicks

Mr. Hicks retired in December 1990 after 30 years of distinguished Federal Service, 28 years of which was providing the Nation with expertise in tides and currents. A graduate of UCLA in 1950 with a major in meteorology, Mr. Hicks received his Masters Degree in physical oceanography from the Scripps Institution of Oceanography of the University of California in 1952. From 1952 to 1962, Mr. Hicks was an Instructor and Assistant Professor of Physical Oceanography at the University of Rhode Island. Since entering the Federal Government (old Coast and Geodetic Survey) in 1962, he has served in research positions; as Chief, Physical Oceanography Division; and Chief, Physical Oceanographic Research Group; until assuming duties as Senior Physical Oceanographer, all in the National Oceanic and Atmospheric Administration (NOAA) and its predecessor organizations. He also has been an Associate Professorial Lecturer in Physical Oceanography at The George Washington University. Mr. Hicks was selected a Fellow in the first class (1964-65) of the Department of Commerce Science and Technology Fellowship Program. In the Program, he studied at the Geophysical Fluid Dynamics Laboratory of NOAA. Mr. Hicks is a 1970 graduate of the senior resident course (College of Naval Warfare) of the U.S. Naval War College in Newport, RI. He received the Silver Medal of the U.S. Department of Commerce for his research on tides, long-period sea-level variations, and tidal datums; and has authored numerous papers in refereed scientific journals on these subjects. His nine military awards from WWII, while serving in the Destroyer USS Conway (DD507), include the Combat-Action Ribbon and the Asiatic-Pacific Campaign Medal with six major engagement stars.

Michael Szabados
Director,
Center for Operational Oceanographic Products and Services

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CHAPTER 1

INTRODUCTION

This book presents an elementary explanation of tides and tidal datums. It is written to explain the natural phenomenon of tides in terms and concepts readily understandable by students as well as those in all walks of life merely wishing to be enriched by additional knowledge of their environment. Also, scientists in fields other than physical oceanography and astronomy may wish to use the book as a primer to the fundamentals of tidal theory. Although not at all essential to a complete understanding of tides, a mathematical development of the tide-generating forces is provided.

Chapter 10 Tidal Datums should be of particular interest to coastal zone managers, coastal engineers, geologists specializing in beach processes, attorneys concerned with boundary litigation, and legislators (at all government levels) representing marine activities and jurisdictions adjacent to the sea. Finally, it is hoped that new employees in the tides and tidal datum areas of the National Oceanic and Atmospheric Administration will find this book a very helpful introduction to their careers.

The term tide is often restricted to the vertical rise and fall of the water usually occurring twice in a little more than a day. This rise and fall are best observed on a breakwater or on the piles of a pier. When the water is viewed near the time of its lowest point, called low tide or low water, the extent of marine growth and discoloration indicates the general range of the vertical excursion. The tidal range varies, from place to place over time, from almost nothing to many feet.

With tides on a sloping beach, the water moves up the beach landward and down the beach seaward. Similarly, but on a grander scale, it moves inland and then seaward across the sloping mud flats and marshes of estuaries as the tide alternately rises and falls.

The horizontal component of this phenomenon is called the tidal current. It is best seen at an inlet connecting the ocean with a barrier sound. The tidal current floods and then ebbs with slack waters in between. At any particular location, the high and low tides; together with the floods, ebbs, and slacks of the tidal current; have a particular sequence of occurrence with near constant time intervals between each other.

The tide (in its restricted sense) and tidal current are both integral parts of one major phenomenon that will be called hereafter, the tide. Tides should be thought of as being in the form of waves. These waves are thousands of miles in length. Their crests are the high tides, their troughs, the low tides, and the horizontal component of the water particles that make up the wave, the tidal currents. To complicate the matter, these waves combine to reinforce or interfere with each other in varying amounts, partially contributing to the wide differences in tidal characteristics as actually observed.

The tide is fundamentally caused by gravitational interactions between the sun, moon, and earth. These interactions of the gravitational forces are the same as those causing the moon and earth to remain in their respective orbits.

It is often said of science that the ability to predict a natural event is indicative of understanding. Since tides are one of the most accurately predictable natural phenomena, it follows from the axiom that the tide is truly understood. Nothing could be further from the truth. The sciences of astronomy and geophysics provide very accurate quantitative determinations of the tide-generating forces on the earth. The science of physical oceanography provides a detailed understanding of wave dynamics and the response of the ocean to the tide-generating forces.

The Center for Operational Oceanographic Products and Services (CO-OPS) of the National Ocean Service (NOS), National Oceanic and Atmospheric Administration (NOAA) provides very accurate measurements of the tide over time at numerous locations throughout the United States and its territories. But, between the fundamental tide producing forces and the observed tide at a particular place and time, there is a vast area of numerous unknowns that will be referred to as terrestrial factors. Some of the factors that the unknowns are associated with are:

1. the restrictive depths of the oceans not allowing the generated tidal wave to be in equilibrium with the rotation of the earth,
2. irregular ocean depths over which the waves must travel,
3. reflections and interactions of the waves from irregularly shaped continents,
4. bottom friction,
5. turbulence, and
6. viscosity of the water.

In addition to the terrestrial factors mentioned above there are also meteorological factors which, at some locations, cause large differences between the observed and predicted tide. Weather events such as hurricanes and tropical storms can produce significant differences between predicted and observed water levels. Wind, rain (associated with river runoff), and barometric pressure, even when not associated with a storm event, can also cause major differences between observed and predicted water levels.

How is it, then, that the physical oceanographers at NOAA are able to predict the tide with great accuracy? By employing an empirical method the tide-generating forces can be related to the observed tide with sets of numerical constants unique to a particular location. Essentially, these sets of constants embody the terrestrial factors partially listed above. With astronomical information readily available, extrapolations can be made for predictions years in advance using the same sets of constants. Why does it work? First, because it is correct that the tide is fundamentally caused by the gravitational interactions between the sun, moon, and earth. Second, because the tide can be observed with great accuracy at a particular location over time. And third, because the earth happens to be a *very consistent responder* to these tide-generating forces, even though all the answers associated with the processes are not known. However, incorporating meteorological factors is not feasible for predictions at the desired degree of accuracy. Only the astronomic tides are considered in this book.

The analysis and prediction of tides are covered in the succeeding chapters. This is not for the purpose of teaching one how to predict the tides, but rather for the understanding that will be achieved in breaking down the total system into its simple components and then recombining these components into the realistic ocean tide.

CHAPTER 2

GRAVITATIONAL ATTRACTION

In the ordinary course of human experience, gravitational attraction seems to be a fundamental and universal force. It maintains the planets and their moons in orbit and gives rise to the gravity we feel on earth. Gravitational attraction is a basic force in everyday Newtonian physics. Newtonian physics describes motion extremely well, providing the motion does not approach the speed of light and is not in close proximity to a very large body, like the sun. For example, the motion of the planet Mercury, closest to the sun, is not described perfectly by this physics. However, as with the vast amount of observed and experienced motion, tides are best described and understood through Newtonian physics.

Gravitational attraction possesses several characteristics of considerable interest. It is well known that objects of differing weights fall at the same speed in a vacuum. Also, that gravitation is bidirectional. That is, the moon attracts the earth as the earth attracts the moon.

More obscure characteristics, described thoroughly by Heyl (1954), are that observations of comets show that gravitation is not affected by changes in temperature. Pendulum experiments with different bobs show no differences, whether made of terrestrial or meteoritic material, solid or liquid substances, radioactive or natural items, or the same substance before or after a chemical reaction. Similarly, crystalline items of the same weight in any orientation and magnetized and non-magnetized substances show no differences.

Gravitation cannot be screened either. Astronomers have looked for the cumulative effects of minute perturbations in the orbit of the moon when it is directly behind the earth in a total lunar eclipse, but to no avail.

Newton had no suggestion as to the fundamental cause of gravitation. Einstein related it to the warping of the space coordinate system in the presence of large masses. The bending of starlight as it passes close to the sun is one of the verifications. An analogy, although not entirely satisfactory, is a large weight placed in the center of a pond with a frozen surface. The weight will depress the ice such that a bowling ball, thrown at just the right speed and to just miss the weight, will revolve around it in an ellipse simulating the effect of gravitational attraction. The idea here is that the geometric distortion of the ice is analogous to the warping of the space coordinate system.

Gravitational attraction is directly proportional to the product of the masses involved and inversely proportional to the square of the distance between the centers of mass. The square in the denominator means that distance becomes quite important. As will be seen in subsequent chapters, the development of the tide-generating forces involves a cube in the denominator, making distance *very* important in tides.

The words “tide,” as in “high tide,” and “water,” as in “high water,” have been used interchangeably in the historical development of tidal activity here in the United States. In the last few decades, “water” has been favored. Whenever ocean elevation measurements are made, the highs and lows *include* the astronomic tides and all other effects such as those caused by the winds and ocean currents. Therefore, “water” is the most appropriate term. In analyzing, studying, and predicting, however, “tide” is the most appropriate since the astronomic tide is the element that we are able to predict with the greatest accuracy and reliability. Recently, the term “water level tidal prediction” has been introduced by CO-OPS in an attempt to solve this semantic problem.

Here, “tide” will be used throughout since the subject is astronomic tides. However, “water” will be used in such terms as “mean lower low water” in Chapter 10 Tidal Datums. The latter use is because tidal datums are precisely defined terms that have been recognized in law as they relate to coastal and marine boundaries (ownership, jurisdiction, and legal responsibility).

CHAPTER 3

THE SUN-EARTH AND MOON-EARTH SYSTEMS

To best understand the tide, it is necessary to study each astronomical motion, together with its associated tide producing forces, separately. Furthermore, it is necessary to study each as a simple, idealized motion. Later, these separate, simple, idealized motions will be combined to form a realistic description of the astronomic phenomena and the tides for which they are responsible.

The Sun-Earth System

The sun and the earth revolve around their common center of mass in one year. Although the orbits are elliptical and the earth is inclined to the ecliptic (its plane of revolution around the sun), these facts are not considered at this time. The mean distance between the sun and the earth is about 92,897,400 miles. The center of mass of the sun-earth system is located about 280 miles (mean) from the center of the sun. Therefore, it might be easier to visualize the sun as wobbling around this point.

Primary development

The tide-generating forces may be developed by considering *differences* in the gravitational attraction of the sun at various locations on the earth. At the center of mass of the earth the gravitational attraction must be a value just necessary to keep the earth in its orbit around the sun. This is F_2 in Figure 1. At a point on the side of the earth facing the sun, the gravitational force, F_1, is greater than at the center because it is closer to the sun. Since water is more fluid than the earth, the water distends toward the sun. At a point on the side of the earth away from the sun, the gravitational force, F_3, is less than at the center because it is further away from the sun. Thus, the water distends away from the sun. Quantitatively, the tide-generating forces are the differences between the gravitational attraction at the center of the earth and the gravitational attractions at the points of interest, as altered by distances from the sun.

$D_1 = F_1 - F_2$ directed toward the sun

$D_3 = F_2 - F_3$ directed away from the sun

The same development can be used for all points on the earth. The ends of the difference vectors, D, will form an envelope of the tide-generating forces. It is a symmetrical egg-shaped distention that many

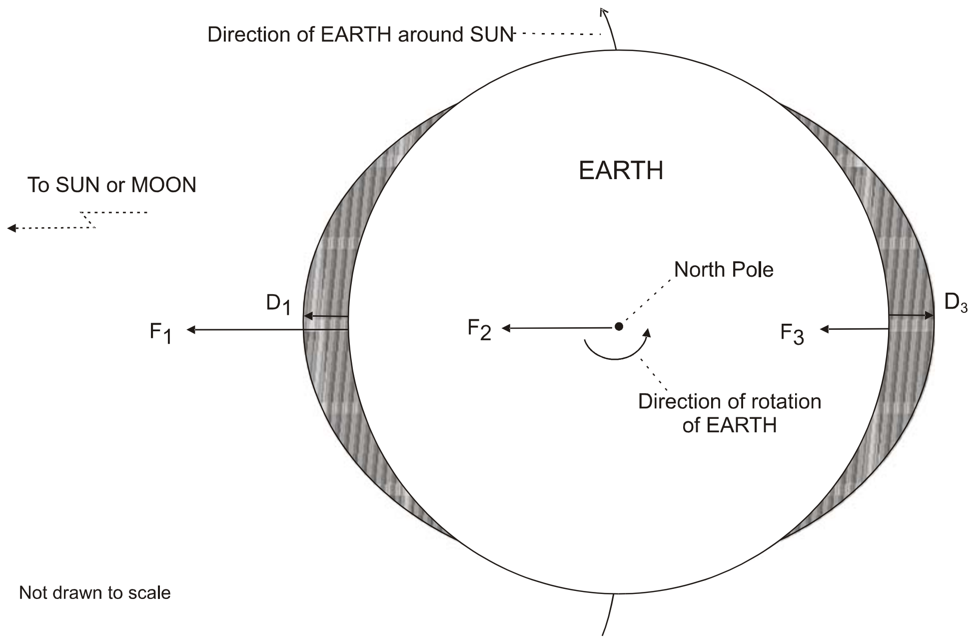

Figure 1. Envelope of tide generating forces

refer to as the tidal bulge of the ocean waters. It is not. Again, it is the envelope of the tide-generating forces - the tide potential. It is the distention that the tide-generating forces are *trying* to make in the ocean waters but never realize because of the factors mentioned in Chapter 1, i.e., restriction of wave speed, irregular depths, presence of continents, bottom friction, turbulence, and viscosity. It is frequently referred to as the equilibrium tide.

Looking down on the North Pole in Figure 1, the earth spins counter clockwise in one solar day. The mean solar day is the period of rotation of the earth on its own axis (spin) referenced to the mean sun and is divided into 24 solar hours. Starting with the point on the equator toward the sun, the point experiences a maximum in the solar tide-generating force; six hours later, a minimum; then a maximum at twelve hours; a minimum again at eighteen hours; and finally back to the maximum at the conclusion of the solar day. Note that in one solar day there are *two* complete tide-generating force cycles. The distended symmetrical egg-shaped envelope of the tide-generating forces stays lined up with the sun as the earth and sun go around their common center of mass each year. The earth spins (slips) once *within* the envelope each solar day.

Alternative development

The earth is maintained in its orbit around the center of mass of the sun-earth system by centripetal acceleration. Centripetal acceleration is provided, in this case, by the gravitational attraction between the sun and earth. Centrifugal force, a fictitious or apparent force, may be introduced for convenience. The concept of centrifugal force is derived from the inertial tendency of the earth to go flying off into space tangentially to its orbit. When considered over 360°, it may be represented by a vector directed away from the center of mass of the sun-earth system.

Centrifugal forces are the same throughout the earth. This is realized by remembering to take only one motion at a time and to consider irrotational motion. In irrotational motion, the earth does not spin on its axis even once during the year. Thus, only the revolution of the earth around its common center of mass with the sun takes place. Therefore, any location on the earth has the same radius as any other and the centrifugal forces at all points on the earth are equal.

Gravitational forces differ throughout the earth. The closer a point on the earth is to the sun, the greater the force. This is because gravitational force varies inversely as the square of the distance between the sun and earth. Since the earth does not fall into the sun or fly off into space (significantly during historic times), the gravitational force must be balanced by the centrifugal force (equal but oppositely directed vectors) at the center of mass of the earth. On the side of the earth facing the sun, the gravitational force is greater than the centrifugal causing a tide producing force directed toward the sun. On the side of the earth away from the sun, the gravitational force is less than the centrifugal causing a tide producing force directed away from the sun.

It is unimportant which of the above developments is used for understanding. The two ways of thinking about it are the same and yield the same results. However, the primary development will continue to be used for continuity.

The Moon-Earth System

Primary development

The moon and the earth revolve around their common center of mass in one month. Although their orbits are elliptical and the earth’s equator and moon’s orbit are inclined to the ecliptic, these facts are not considered at this time. The mean distance between the moon and the earth is about 238,860 miles. The center of mass of the moon-earth system is located within the earth about 2895 miles (mean) from the center of mass of the earth, always toward the moon.

Figure 1 shows the gravitational force vectors, F, and the difference vectors, D. They are essentially the same as for the sun-earth system except for magnitude. The same development can be used for all points on the earth as previously explained for the sun-earth system.

Looking down on the North Pole in Figure 1, the earth spins counter clockwise in one lunar day. The mean lunar day is the period of rotation of the earth on its own axis referenced to the moon and is 24.84 solar hours in length. Starting with the point on the equator toward the moon, the point experiences a maximum in the lunar tide-generating force; 6.21 solar hours later, a minimum; then a maximum at 12.42 solar hours; a minimum again at 18.63 hours; and finally back to the maximum at the conclusion of the lunar day. Note that in one lunar day there are *two* complete tide-generating force cycles, just as there were for the sun. However, the solar cycle took 24 hours and the lunar, 24.84. The distended symmetrical egg-shaped envelope of the lunar tide-generating forces stays lined up with the moon as the earth and moon go around their common center of mass each month. The earth spins (slips) once *within* the envelope each lunar day.

Alternative development

The alternative development of the tide-generating forces for the moon-earth system is the same as for the sun-earth system. However, the center of mass of the moon-earth system is deep within the earth. By taking one motion at a time and considering irrotational motion, the earth wobbles around this point. Since the earth does not rotate (spin) during the month with irrotational motion, every point on the earth describes a circle of the same radius. Visualization of the centrifugal force on the side of the earth facing the moon is difficult, at first. Irrotational motion and consideration of only the monthly revolution are the keys. The centrifugal force vector on the side of the earth facing the moon is directed *away* from the moon like all the other points on the earth.

CHAPTER 4

SPRING AND NEAP, DISTANCE AND DECLINATION

Spring and Neap Tides

The range of tide, defined as the vertical difference in height between consecutive high and low tides, varies from place to place and also varies over time. The combination of the solar and lunar envelopes during the synodic month causes spring tides and neap tides. The synodic month is referenced to the sun (or phase of moon) and is 29.530588 days in length.

By combining the sun-earth system and the moon-earth system as outlined in Chapter 3, spring and neap tides are developed. Spring tides have nothing to do with springtime. They are tides with ranges greater than the average monthly range. Spring tides occur twice each synodic month due to the approximate alignment of the sun, moon, and earth. Neap tides are tides with ranges less than the average monthly range. They occur twice each synodic month due to the sun, earth, and moon forming right angles (approximately) to each other.

Colloquially, the tide is said to "follow the moon" since it is 2.16 times more influential than the sun in causing the tide. It is probably more descriptive to say that the tide, in most locations, is controlled by the lunar tide-generating forces *as modified* by the solar.

When the moon is new (on the side of the earth toward the sun) or full (on the side of the earth away from the sun), the tide-generating forces of the sun and moon are aligned. In Figure 2, the envelope of the tide-generating forces of the sun augments the envelope of the moon. The high tides of the solar envelope occur at the same time as the high tides of the lunar. This increases the height of the composite high tides. Likewise, the low tides of the solar envelope occur at the low tides of the lunar. This decreases the height of the composite low tides. Therefore, larger than average tidal ranges occur, called spring tides.

When the moon is in its first or third quarter, however, the tide-generating forces of the sun are at right angles to those of the moon. In Figure 3, the envelope of the tide-generating forces of the sun is shown to conflict with the force envelope of the moon. The low tides of the solar envelope occur at the times of the high tides of the lunar. This reduces the height of the composite high tides. Likewise, the high tides of the solar

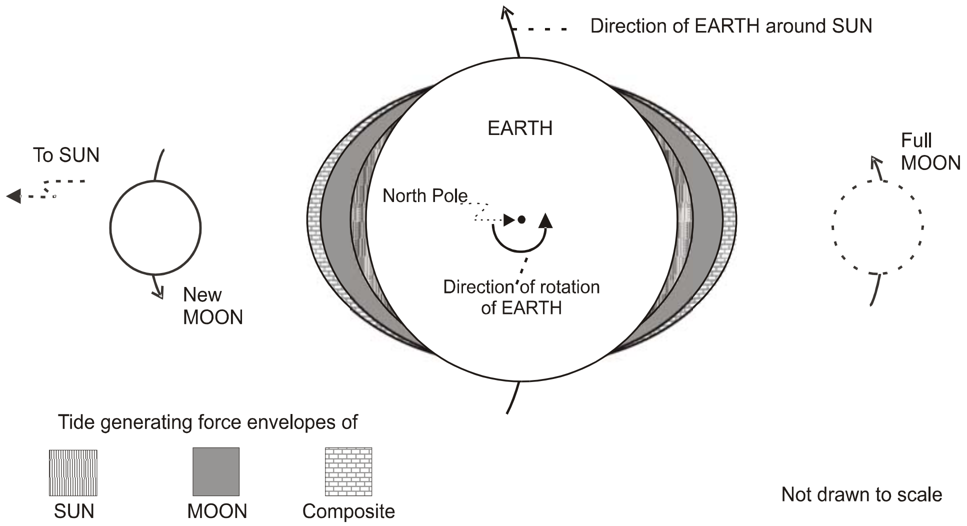

Figure 2. Spring tides

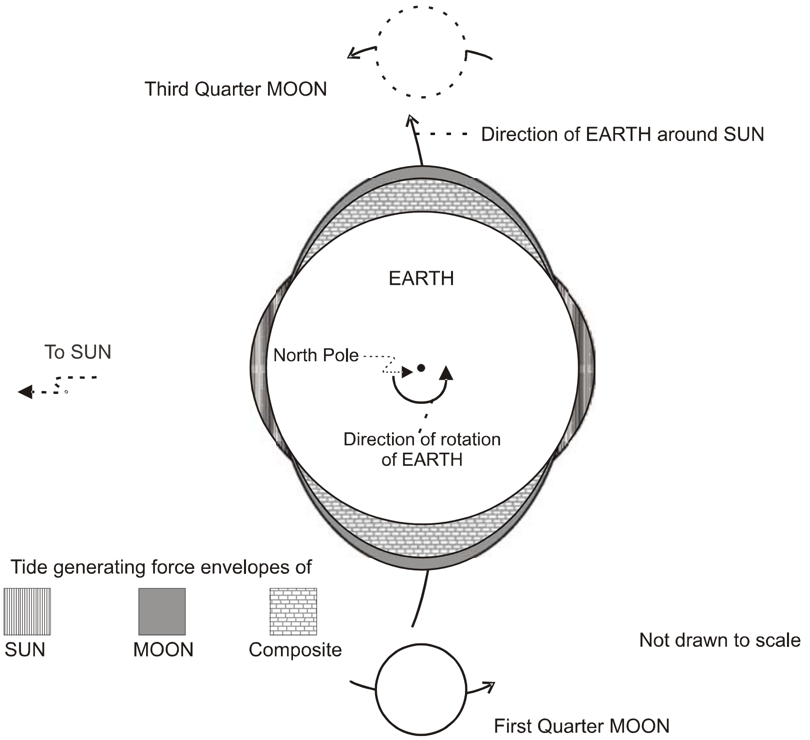

Figure 3. Neap tides

envelope occur at the low tides of the lunar. This increases the height of the composite low tides. Therefore, smaller than average tidal ranges occur, called neap tides. The change in range between spring and neap tides amounts to about 20%.

In addition to changes in tidal range, the spring-neap tide phenomenon also causes changes in the time of occurrence of the high and low tide phases of the lunar tide. As the moon goes from new to first quarter or full to third quarter, the solar envelope is behind the lunar such that the composite is retarded. Thus, as the earth spins, the high and low tide phases of the composite occur earlier than with just the lunar envelope. This is called priming. Priming is shown in Figure 4.

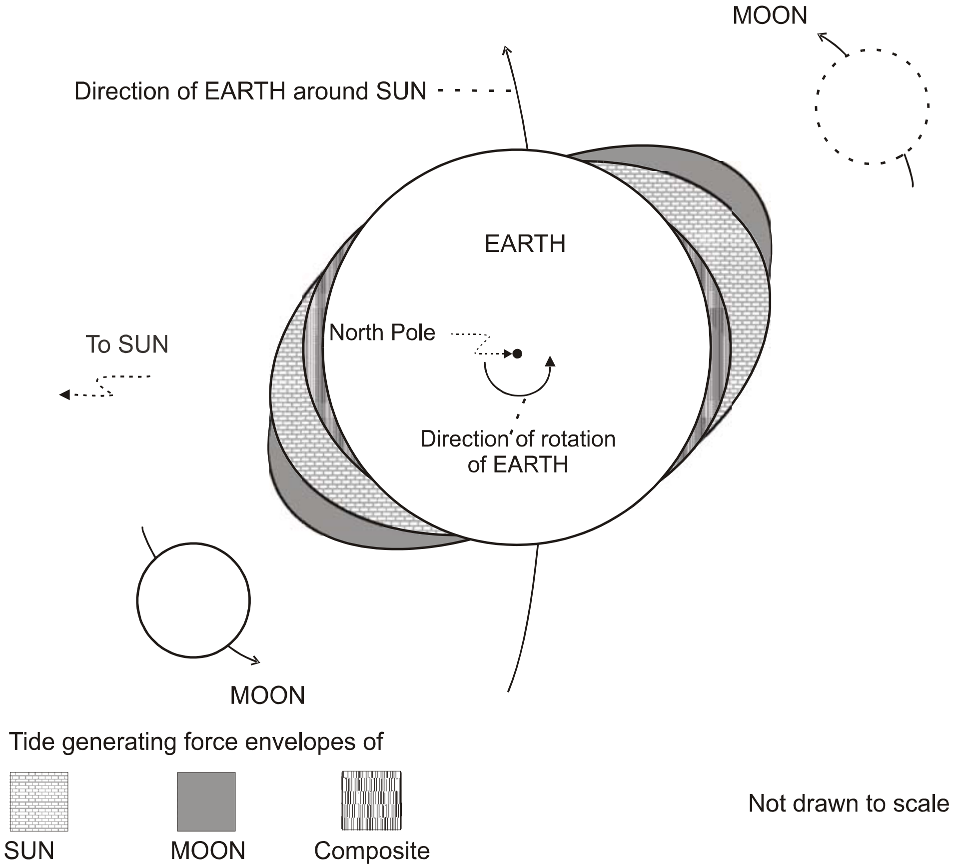

Figure 4. Priming

Similarly, as the moon goes from first quarter to full or third quarter to new, the solar envelope is ahead of the lunar such that the composite is accelerated. Thus, as the earth spins, the high and low tide phases of the composite envelope occur later than with just the lunar envelope. This is called lagging and is illustrated in Figure 5.

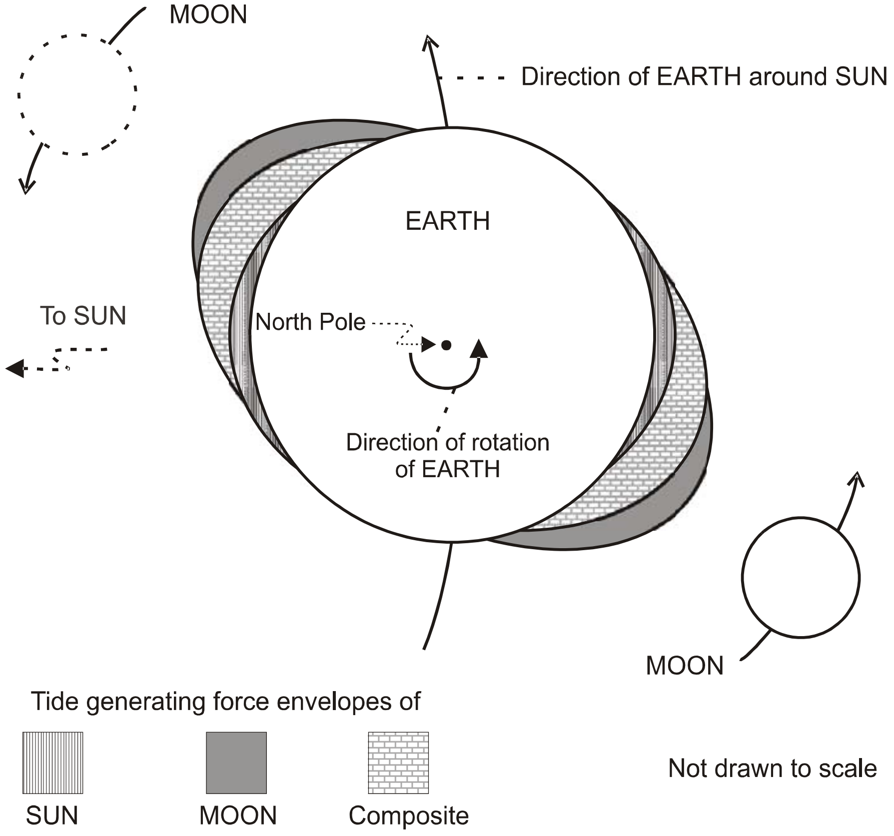

Figure 5. Lagging

Distance

The sun and earth revolve around their common center of mass in elliptical, rather than circular, orbits. When the earth is closest to the sun, the earth is said to be in perihelion. When the earth is farthest from the sun, it is said to be in aphelion. Perihelion to perihelion occurs in one anomalistic (a-nom-a-lis-tic) year of 365.2596 days. Perihelion occurs on January 3rd [dates are for the year 2007, Coordinated Universal Time

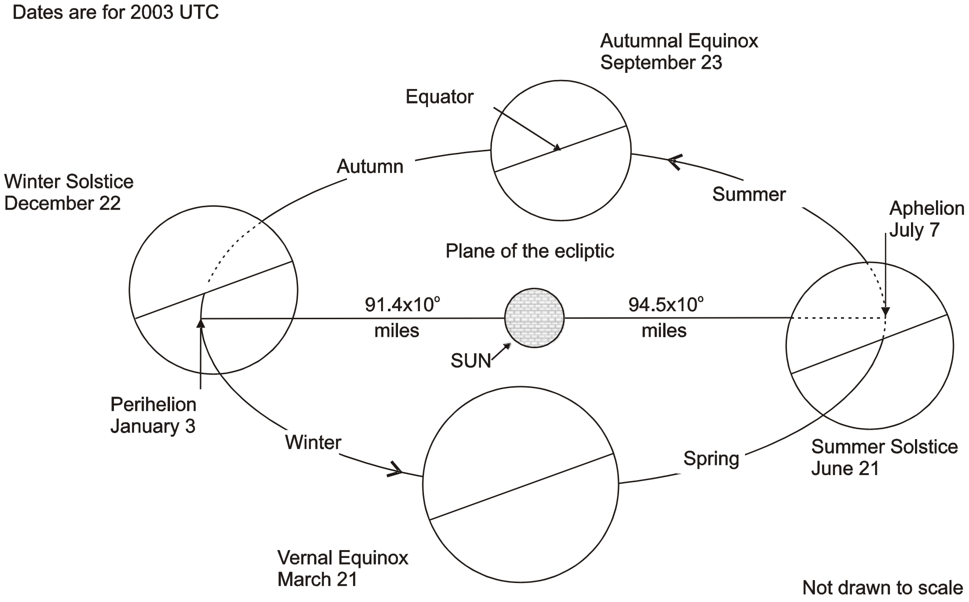

Figure 6. the sun-earth system

(UTC)], with the distance being about 91,400,005 miles. Aphelion occurs on July 7th, the distance being approximately 94,512,258 miles (Figure 6). It will be shown that the tide-generating forces are inversely proportional to the cube of the distances between the two bodies. Therefore, when the earth is in perihelion, the tide-generating forces are larger than average and greater than average ranges in the tide are experienced. In aphelion, smaller than average ranges occur.

The moon and earth also revolve around their common center of mass in elliptical orbits. When the moon is closest to the earth, the moon is said to be in perigee. When the moon is farthest away, it is in apogee. Perigee to perigee occurs in one anomalistic month of 27.554,550 days. At perigee the two bodies are about 221,000 miles apart while at apogee they are about 253,000 miles away from each other (Figure 7). When the moon is in perigee, the tide-generating forces are larger than average and greater than average ranges in the tide occur. The opposite is true when the moon is in apogee.

Declination

The ecliptic is the plane generated by the earth as it revolves around the center of mass of the sun-earth system. The axis of rotation (spin) of the earth is inclined 23.452° to the ecliptic. This gives rise to the seasons experienced on earth. The cycle of the seasons is the tropical year of

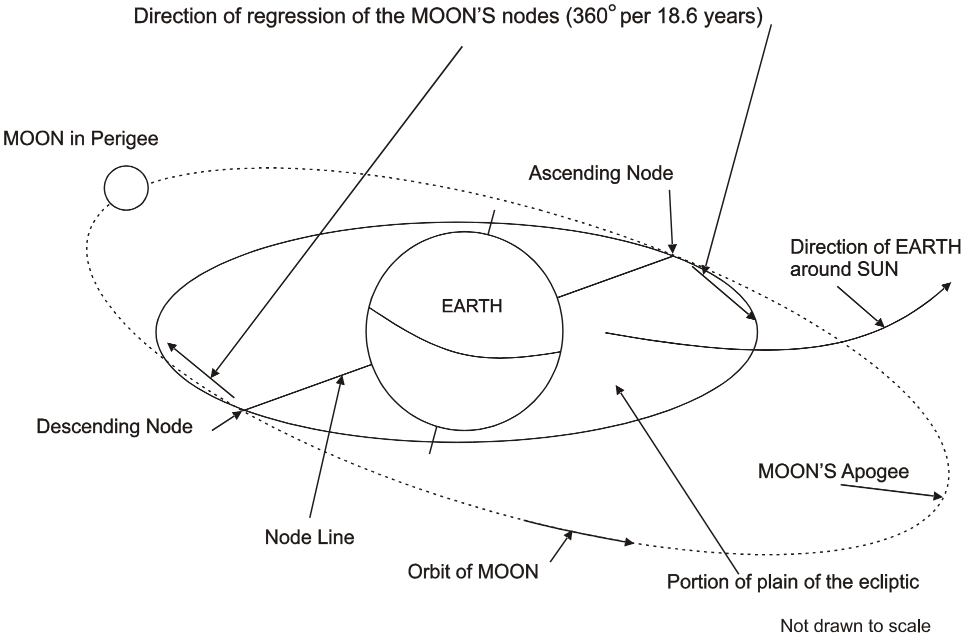

Figure 7. the moon-earth system

365.2422 days. It is referenced to the vernal equinox (direction when the apparent sun passes from south to north over the earth's equator).

On June 21st (2007, UTC) the northern hemisphere is tipped directly toward the sun (Figure 6). The sun is said to be at its maximum northern declination (angular height above the equator of 23.452°). This is the summer solstice and the beginning of the season of summer in the northern hemisphere.

Three months later, on September 23rd, the earth (still tipped in the same direction relative to a star) has moved one-quarter of the way around the ecliptic such that the sun is now over the equator (zero declination). This is the autumnal equinox signaling the beginning of autumn for the northern hemisphere.

Winter for the northern hemisphere begins on December 22nd, the winter solstice, when the sun is at its maximum declination south. The vernal equinox, ushering in spring for the northern hemisphere, occurs on March 21st when the sun passes over the equator on its apparent trip north. The seasons are reversed, of course, in the southern hemisphere.

The distended symmetrical egg-shaped envelope of the solar tide-generating forces remains lined up with the sun as described in Chapter 3. However, with the earth tilted, the forces are asymmetrical with respect to the equator as the earth progresses through the seasons. The differences

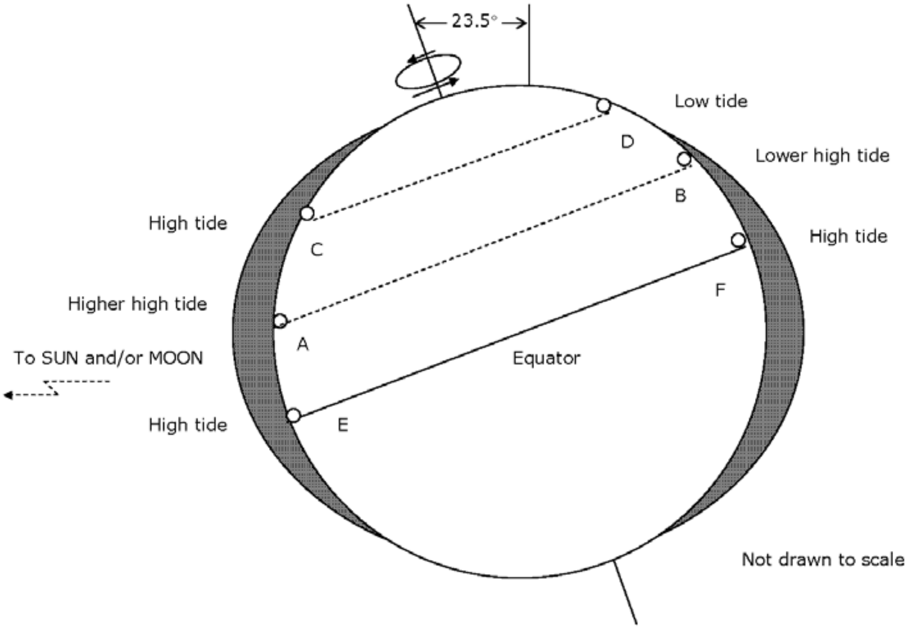

Figure 8. Diurnal inequality

between the two daily maxima of the tide-generating forces for various locations at the summer solstice are shown in Figure 8. At A the high tide is higher than at B, 12 hours later. This is called diurnal inequality in the semidiurnal tide. At the latitude of CD, there is only one high tide per day. Tides from this situation are known as diurnal tides. At the equator, EF, the high tides are equal, giving a typical semidiurnal tide. Although not shown in this figure, the forces are symmetrical with respect to the equator at the equinoxes and, of course, no solar diurnal inequality occurs from this effect at these times.

The plane of the orbit of the moon is inclined 5.145° to the ecliptic – not our equator (Figure 7). The two places where the moon crosses the ecliptic are called the moon's nodes. The place where it crosses the ecliptic from south to north is known as the ascending node. Where it crosses from north to south - the descending node. The lunar declinational effect also causes diurnal inequality in the tides. Because it is due to the moon, the effect is greater than that of the sun and its cycle is the nodical month (referenced to the moon's ascending node) of 27.212220 days.

Looking down on the northern hemisphere, the earth revolves around the sun counterclockwise, the moon revolves around the earth counterclockwise, and the earth rotates counterclockwise. The moon's nodes, however, move clockwise (or westward) on the ecliptic (Figure 7).

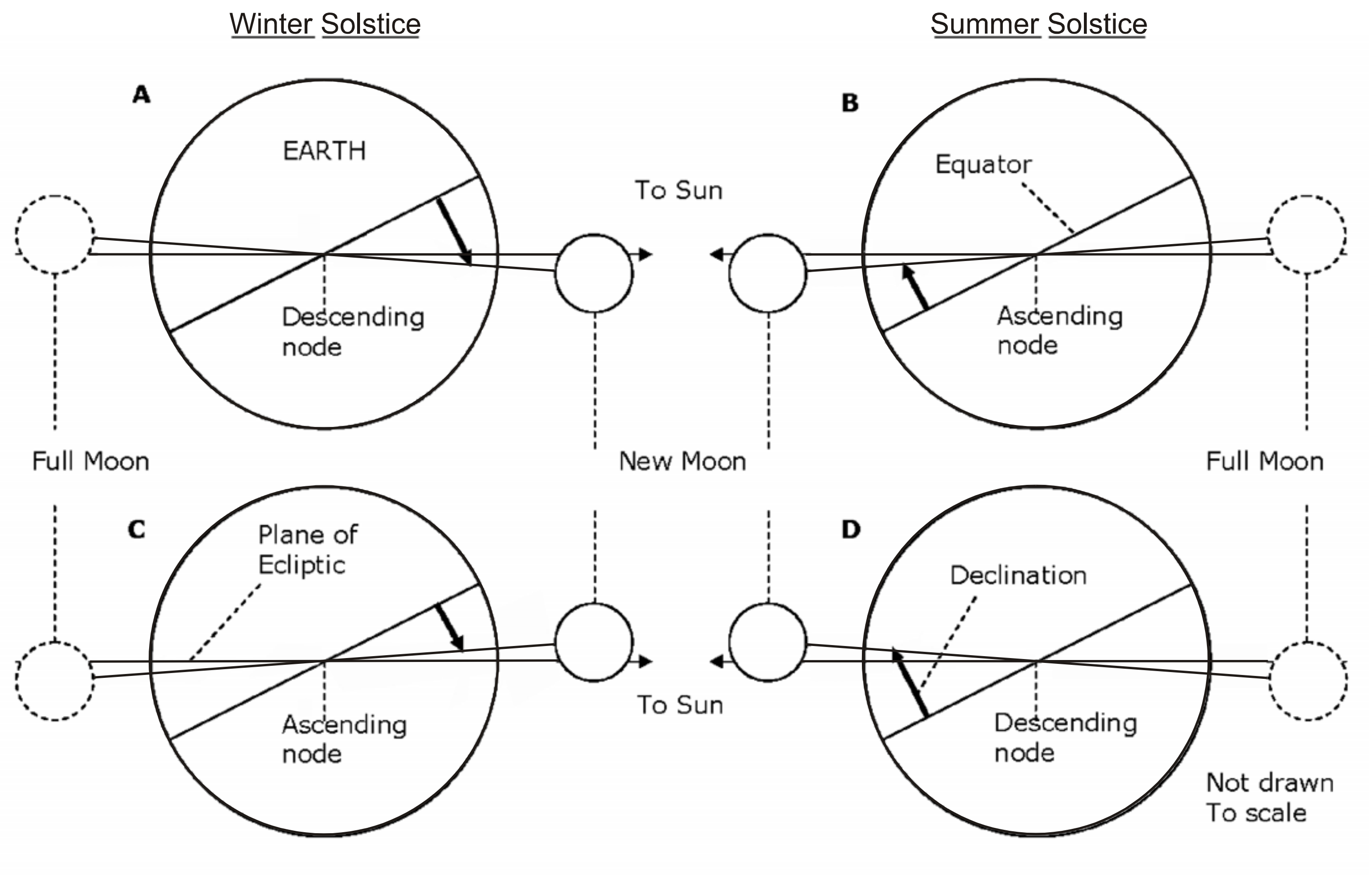

Figure 9. Lunar node positions for diurnal inequality

This is called the regression of the moon's nodes and it takes 18.61 years for them to go completely around and back to their original position. It is best understood by visualizing the plane of the orbit of the moon *pivoting* on the plane of the orbit of the earth (the ecliptic). The phenomenon causes the moon to vary in its maximum monthly declination (distance north and south of the earth's equator) during the 18.61-year nodal cycle. The maximum monthly declination of the moon will range between:

23.452° + 5.145° = 28.597°, and

23.452° – 5.145° = 18.307°.

During the 18.61-year nodal cycle, times will occur when a line connecting the moon's nodes (node line, Figure 7) will be at about right angles to a line connecting the earth and sun. When this happens near the winter solstice with the moon below the ecliptic on the sun side, maximum monthly declinations will build up to 28.597° (A in Figure 9) and large diurnal inequalities will occur. This is also true near the summer solstice with the moon above the ecliptic on the sun side, as seen in D. The opposite occurs with the moon below the ecliptic on the sun side near the summer solstice, B, and above the ecliptic on the sun side near the winter solstice, C. In the latter two cases the maximum lunar declination will amount to only 18.307° and relatively small diurnal inequalities will occur.

This 18.61-year nodal cycle results in slowly varying ranges and inequalities of observed tides at tide stations. These changes are significant enough to be measurable and this nodal cycle time-period is used in the computation of accepted tidal datums (see Chapter 10).

CHAPTER 5

TIDE-GENERATING FORCES

The solid earth is not rigid. It undergoes plastic deformation due to the same gravitational interactions as the oceans. Earth tides, as they are called, are relatively small but readily measurable. Although an important and very interesting geophysical phenomenon, they are outside the scope of this book.

Gravitational force, F, is equal to the gravitational constant, G, times the product of the masses of the two bodies divided by the square of the distance between their centers of mass. For the sun and moon, respectively, the gravitational force can be defined as;

$F_s = G(SE) / r^2$ and $F_m = G(ME) / r^2$

where:
S is the mass of the sun,
M is the mass of the moon,
E is the mass of the earth, and
r is the distance between the centers of mass of the earth, and the sun or moon.

$F_{s1} = GSE / (r - \rho)^2$ is the gravitational force at the surface of the earth on the side facing the sun directed toward the sun, where ρ is the equatorial radius of the earth.

$F_{s2} = GSE / r^2$ is the gravitational force at the center of mass of the earth directed toward the sun.

$F_{s3} = GSE / (r + \rho)^2$ is the gravitational force at the surface of the earth on the side away from the sun directed toward the sun.

The differential or tide-generating force is equal to the *differences* between the gravitational force at the center of mass of the earth and the gravitational force at other points on the earth.

The difference at the surface of the earth facing the sun is;

$$F_{s1} - F_{s2} = GSE / (r - \rho)^2 - GSE / r^2$$

$$= GSE[1 / (r - \rho)^2 - 1 / r^2]$$

$$= GSE\{[r^2 - (r - \rho)^2] / [r^2(r - \rho)^2]\}$$

$$= GSE\{[r^2 - r^2 + 2\rho r - \rho^2] / [r^2(r - \rho r/r)^2]\}$$

$= GSE\{[2\rho r - \rho^2] / [r^2[r(1 - \rho/r)]^2]\}$

$= GSE\{[2\rho r - \rho^2] / [r^2 r^2(1 - \rho/r)^2]\}$

$= GSE\{[2\rho r - \rho^2] / [r^4(1 - 0)^2]\}$ since ρ/r is small

$= GSE\{[2\rho r(1 - \rho^2/2\rho r)] / r^4\}$

$= GSE\{[2\rho r(1 - \rho/2r)] / r^4\}$

$= GSE\{[2\rho r(1 - 0)] / r^4\}$ since $\rho/2r$ is very small

$= GSE(2\rho / r^3)$

$= G(2SE\rho / r^3)$

The difference at the surface of the earth away from the sun is;

$F_{s2} - F_{s3} = G(-2SE\rho / r^3)$

or;

$D_s = \pm G(2SE\rho / r^3)$

Similarly the differential or tide-generating force at the surface of the earth facing the moon is;

$F_{m1} - F_{m2} = G(2ME\rho / r^3)$ and, at the surface away from the moon the difference is;

$F_{m2} - F_{m3} = G(-2ME\rho / r^3)$

or;

$D_m = \pm G(2ME\rho / r^3)$

where:

D_s is the differential on the earth due to the sun, and
D_m is the differential on the earth due to the moon at these two locations.

These differentials, one due to the sun, the other due to the moon, are the source of the fundamental tide-generating forces.

With the alternative development of using centrifugal force, the mathematics is the same, since centrifugal force is everywhere equal (but of opposite sign) to the gravitational force at the center of mass of the earth.

It is to be especially noted that although the gravitational force varies inversely as the square of the distance between the two bodies, the tide-generating force varies inversely as the ***cube*** *of the distance*. This is very important since it accounts for the moon being 2.16 times more influential in causing the tide than the sun. This is shown by;

$$D_m / D_s = \pm (2ME\rho / r^3_m) \text{ x } (r^3_s / 2SE\rho)$$

where:
r_m is the distance (in miles) between the center of mass of the earth and center of mass of the moon, and
r_s is the distance (in miles) between the center of mass of the earth and the center of mass of the sun, so;

$$D_m / D_s = (M / S) \text{ x } (r^3_s / r^3_m)$$

If the mass of the moon is taken as one, then;

$$\begin{aligned} D_m / D_s &= (1 / 27{,}069{,}106) \text{ x } (92{,}897{,}400^3 / 238{,}860^3) \\ &\cong (1 / 27 \text{ x } 10^6) \text{ x } [(93 \text{ x } 10^6)^3 / (24 \text{ x } 10^4)^3] \\ &\cong (1 / 27 \text{ x } 10^6) \text{ x } (804{,}357 \text{ x } 10^{18} / 13{,}824 \text{ x } 10^{12}) \\ &\cong 2.16 \end{aligned}$$

Thus, although the sun is 27 x 10^6 times greater in mass than the moon and the moon is only 389 times nearer to the earth than the sun, the moon is still 2.16 times more influential in causing the tide than the sun (or, the sun's influence is .46 that of the moon). Again, this is because the tide-generating force varies inversely as the ***cube*** of the distance. In the tide-generating force equation, the cube of 93 x 10^6 is a very large number in the denominator. It dominates the mass in the numerator to make a relatively small value of the ratio.

For simplicity, the tide-generating forces at the points on the earth directly beneath and away from the sun or moon have been developed. Forces act in varying amounts throughout the surface of the earth forming the envelopes previously described and may be similarly computed. However, the angles formed by the force vectors directed toward the centers of mass of the sun and moon must be considered.

While the solar and lunar envelopes are thought of as representing the actual ocean waters, another very important factor must be recognized. The components of the tide-generating forces acting tangentially along the water surface turn out to be the most important. Just as it is easier to slide a bucket of water across a floor rather than to lift it, the horizontal tractive components move the waters toward the points directly beneath and away from the sun or moon far more effectively than the vertical components can lift them. These tractive forces are most responsible for trying to form the ocean into the symmetrical egg-shaped distensions (the tide potential, the equilibrium tide). They reach their maximums in rings 45° from the points directly beneath and away from the sun or moon.

These tide producing forces set the waters of the ocean basins into periodic motion. The open ocean tides then progress onto the continental shelves and up into the various estuaries and bays where they undergo even more complex transformation. The actual response of the ocean basins to these tide producing forces can be quite complex and is discussed in the following chapters.

CHAPTER 6

WAVE MOTION

Since tides are in wave form, it is essential to examine the mechanism of wave propagation in the ocean. This is particularly true for an understanding of tidal currents and tidal bores.

The crest is the top or high point of a wave. The trough is the valley or low point of a wave. Waves are described by their:

1. Height, H, the vertical distance between the level of a crest and the level of a trough (amplitude, A, is one half the height),
2. Length, L, the horizontal distance from one crest to the next,
3. Period, T, the time interval between the occurrence of two successive crests at a fixed point, and
4. Speed (celerity), C, the length divided by the period, C = L / T, expressed in distance per unit time.

The internal workings of waves and their transformations in shallow water are best understood by following swell from the open ocean to shore. Swell are wind waves that have moved out of the wind generating area or have continued after the wind has died down or changed direction in the generating area. Swell are progressive waves. Although each wave *form* moves in distance on the ocean, the water particles that make up the wave do not. Under the crest the water particles making up the wave are going in the same direction as the wave form. Under the trough the particles are going in the opposite direction. Half way between the crest and the preceding trough, the water is moving up to form the next crest one-quarter of a period later. Half way between the crest and the following trough, the water is going down to form the trough one-quarter of a period later. Thus, over one wave period, the particles making up the wave describe circular orbits. Swell in the open ocean are deep water waves. That is, the bottom does not affect them. The orbits decrease logarithmically in diameter with depth (Figure 10).

Actually, there is a slight overlap in these orbits that accounts for a net mass transport of water in the direction of the waves. This, in turn, gives rise to the longshore and rip current system that is necessary to remove the excess water that accumulates in the surf zone. This net mass transport can

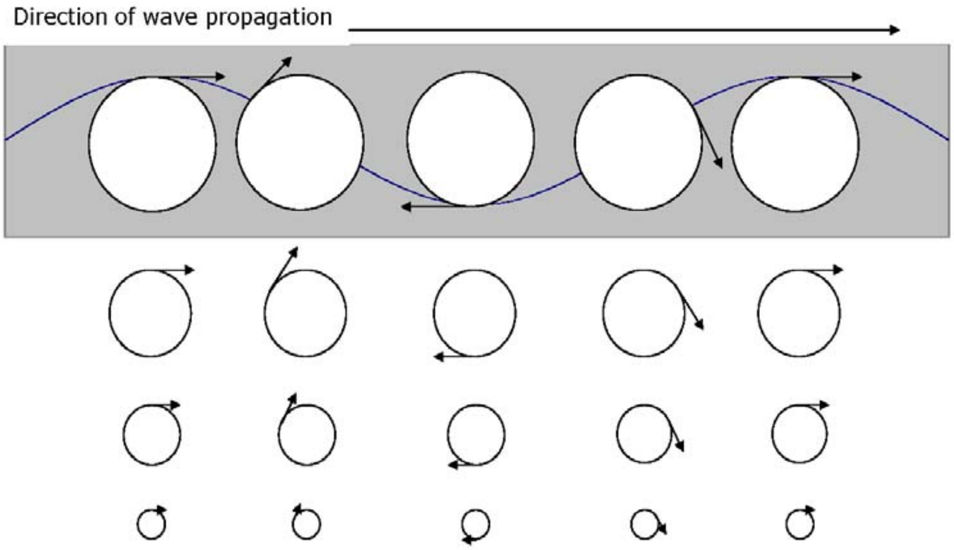

Figure 10. Particle trajectories of progressive wave in deep water

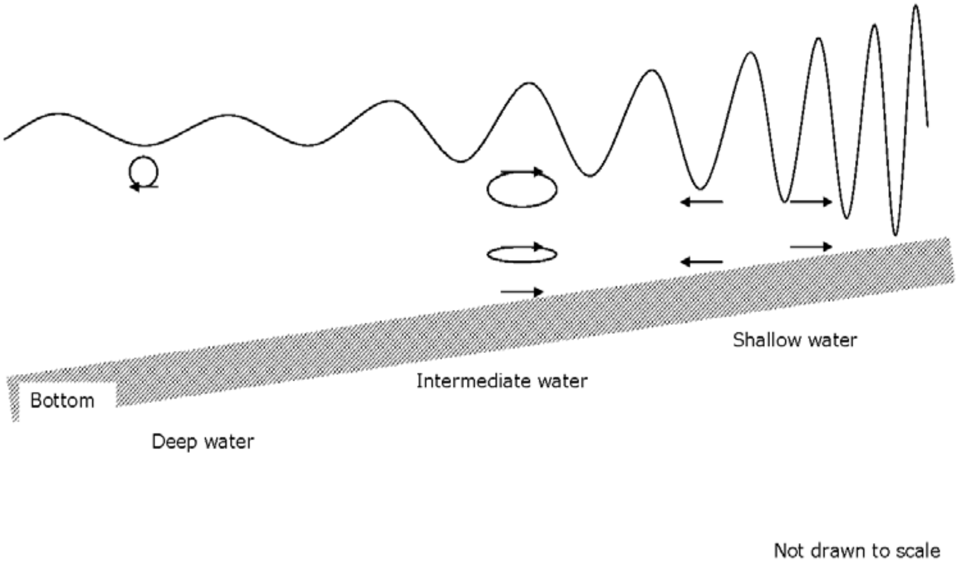

Figure 11. Progressive wave transformation into shallow water

become very significant in high wave conditions accompanying hurricanes and other severe storms.

When swell move into water of depths less than about half of their wave lengths, the circular orbits are restricted by the presence of the bottom. The orbits become elliptical. As the water gets progressively more shoal, the orbits become more and more elliptical until the particulate motion is just back and forth (Figure 11).

Waves in shallow water propagate at a speed equal to the square root of the product of the acceleration of gravity and the depth, $C = \sqrt{gh}$. As a wave moves toward shore, h gets smaller so C becomes less and less. T is constant since the same number of waves that enter the surf zone must equal the number that break on the shore (or combine with other waves). Since C is decreasing, T is constant, and $C = L / T$, L is decreasing. Since L is decreasing, H is increasing (the accordion effect, Figure 11). Thus, waves get higher in the surf zone. A time finally arrives when C becomes less than the speed of the particulate motion under the crest. The wave breaks starting at the crest where the particulate speed is greatest. The wave curls over and plunges onto the foreshore. This process is particularly important in understanding tidal bores (Chapter 7).

Tidal waves are always shallow water waves. That is, the length of a tidal wave is very much longer than the depth. Therefore, the particulate motion that makes up the wave is just back and forth. This back and forth particulate motion in a tidal wave is called the tidal current.

Tidal or tide waves are often confused with tsunamis. The word tsunami (meaning great harbor wave in Japanese) is reserved for ocean waves caused by earthquakes, undersea bottom slumping, large meteorite impacts, or undersea volcanic eruptions. The term tidal wave is used for a wave caused by the astronomic tide-generating forces.

CHAPTER 7

TIDAL WAVES IN GULFS AND ESTUARIES

Tides can be described in terms of waves. For a tidal wave, the crest is a high tide and the trough is a low tide. The height of a tidal wave is the range (or tidal range) and the mean of the high and the low tides is called mean tide level. Figure 12 illustrates some of these fundamental characteristics of tidal waves. For a semidiurnal tide (two highs and two lows per tidal day), the time period (T) between successive high waters (crests) is about 12.42 hours. For a diurnal tide (one high and one low tide per tidal day), the time period between successive high waters is about 24.84 hours. In most gulfs and estuaries of the world, the tidal waves are one of two types or a combination of the two. The first is called a progressive wave, the second a standing wave.

In a progressive tidal wave in the open ocean, the horizontal component of the particulate motion under the high tide area is going in the same direction as the tidal wave form and is known as the flood tidal current (or flood tide). Its maximum, directly under a high tide, is called strength of flood. Similarly, in a progressive tidal wave, the horizontal component of the particulate motion under the low tide area is going in the opposite direction as the tidal wave form and is known as the ebb tidal current (or ebb tide). Its maximum, directly under a low tide, is called strength of ebb.

Half way (in time) between high tide and low tide (and low tide and high tide) at the mean tide level, there is no *horizontal* flow. That is, no tidal current. These points are known as slack, ebb begins (or slack before ebb) and slack, flood begins (or slack before flood) respectively. Observed from a fixed point, the time relationships of the tide and tidal current for a progressive tidal wave are shown in Figure 13.

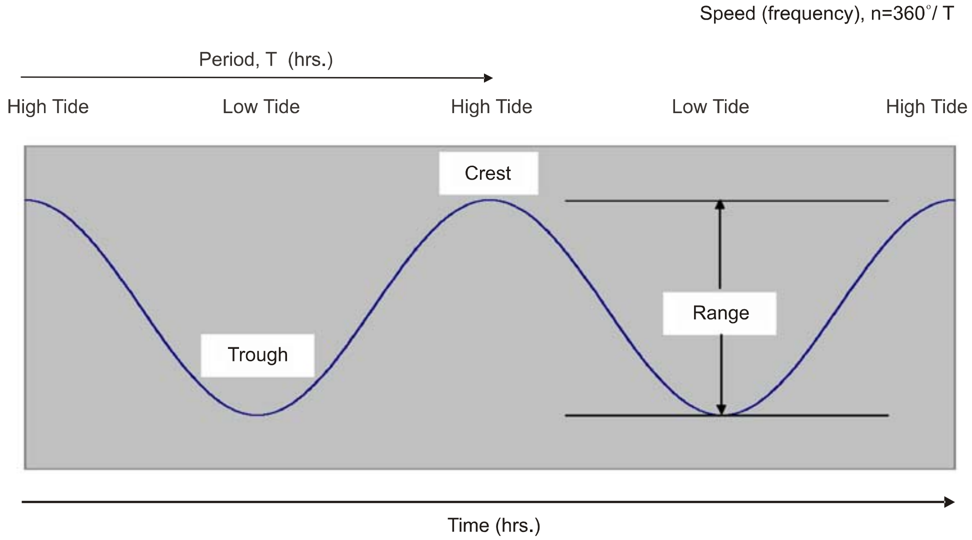

Figure 12. Characteristics of a tidal wave

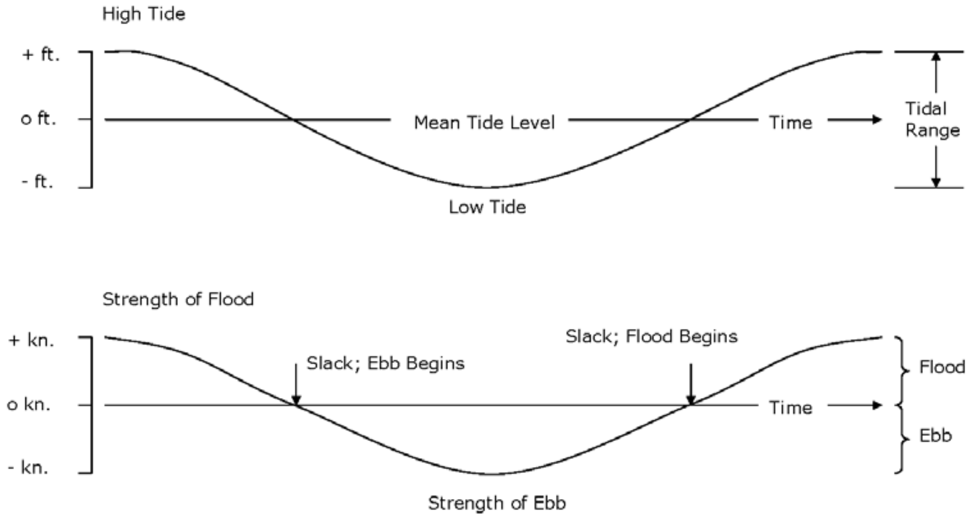

Figure 13. Tidal height and current relationships in a progressive tidal wave

Tides in Estuaries and Bays

When a progressive tidal wave encounters a barrier such as a coast, embayment, head of an estuary, etc., it reflects upon itself to form a standing wave. Figure 14 illustrates the process. Column one shows a progressive tidal wave entering an estuary from the open ocean. Column two shows the reflection from the previous wave moving out of the estuary. Column three shows the resulting combination of the two waves. The presentation assumes complete reflection at the head of the estuary and no attenuation due to friction.

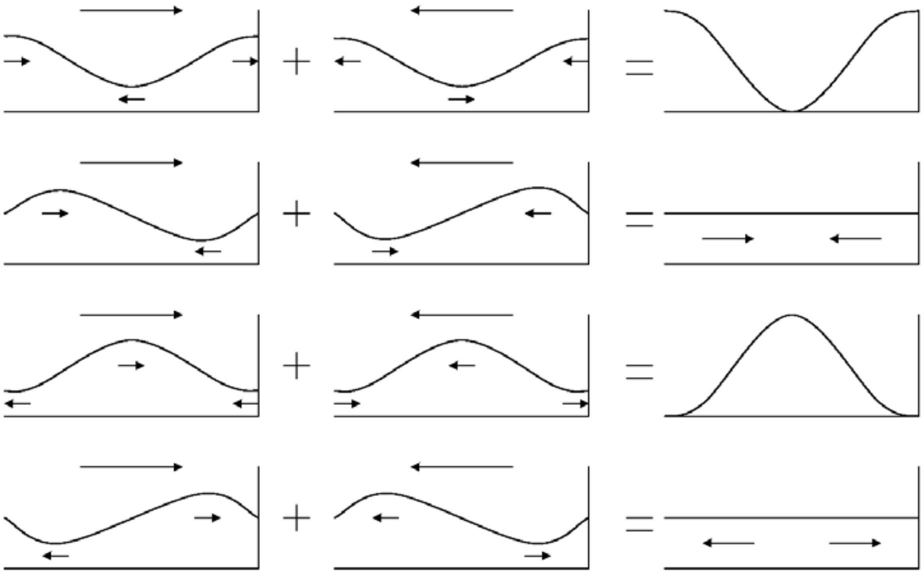

Figure 14. Formation of standing tidal wave from reflection

In the standing tidal wave, the wave form does not progress. Half way (in distance) between the crest and trough (and trough and crest) there is no tidal range. These locations are known as nodes. The wave form merely goes up and down in a standing tidal wave, except at the nodes. There are no horizontal tidal currents under the crests or troughs, called the antinodes. Under the nodes the tidal currents reach a maximum for the entire wave. Choosing a fixed location somewhere between the crest and node near the head, for example, the time relationships of the tide and tidal current are shown in Figure 15.

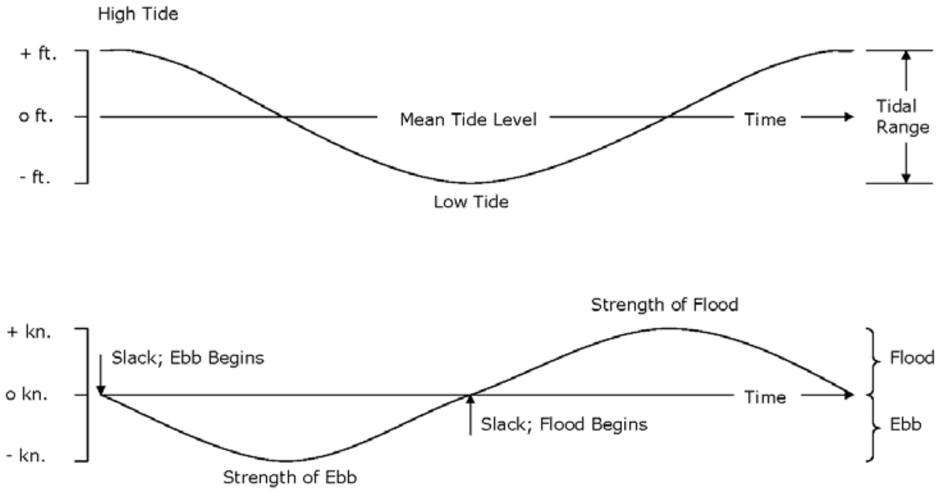

Figure 15. Tidal height and current relationships in a standing tidal wave

Bottom and internal friction complicate the situation. As a tidal wave moves into an estuary, it is attenuated. Its reflection continues to dampen as it moves seaward. The combination of an incoming damped wave and the damped reflection from the previous wave produces a tidal wave whose characteristics are somewhat complicated. It is not a perfect standing wave and tends to have a progressive component. An example is shown in Figure 16.

How much of a tidal wave an estuary can hold is dependent on the estuary's length and depth. From Chapter 6 we know that the formula for the speed of a tidal wave, since it is a shallow water wave, is $C = \sqrt{gh}$. Since $C = L / T$ and T is a constant, L is determined and compared with the length of the estuary. The driving force at the entrance to the estuary is the ocean tide. If a node of the estuary wave happens to be near the entrance, the range of the estuarine tide will be greatly amplified. If an antinode is near, there will be no significant increase from this effect (Figure 17). The tides in the Bay of Fundy are an example of this process. The open Atlantic Ocean tides drive and amplify the tides in the Gulf of Maine. These, in turn, drive and further amplify the tides in the Bay of Fundy. This results in the highest tides of the world, amounting to a spring tide range up to 50.5 feet in Minas Basin.

The Coriolis force affects all motion relative to a rotating earth. In the northern hemisphere, it is directed 90° to the right (looking in the direction of motion). In the southern hemisphere, it is directed 90° to the left. It does not exist for motion along the equator. As a progressive tidal wave enters an estuary, the flood tidal current under the crest is deflected to the right (looking toward the head of the estuary) causing the water to slope down to the left. The ebb tidal current under the trough is deflected to the left (looking toward the head) causing the water to slope down to the right. This accounts for the range of tide being greater on the right side of an estuary than on the left in the northern hemisphere and is known as the Kelvin wave. It also accounts for the tendency of the system to form an amphidromic point – an across estuary nodal line reduced to a point. The motion tends to cause a rocking oscillation of the total estuary and a no-tide point, the amphidromic point, near the center. Actually, in a real embayment, it is probably a minimum tidal-range area due to all the complicating factors such as an irregular embayment and bottom topography, internal and bottom friction and river discharge.

Amphidromic lines, points, or areas also exist in the open ocean. The interactions of various tidal waves in defined oceanic basins cause them to occur. The area near Tahiti, for example, is a well-known one.

The damped incoming tidal wave and its combination with the damped reflection of the previous wave, together with the Coriolis force and amplification, usually result in a standing type tidal wave with a progressive component and a slightly greater tidal range on the right. The amount of wave contained in the estuary (location of node) and the driving force from the open ocean tide determine, in general, the tidal ranges, strengths of tidal currents, and time relationships throughout the estuary.

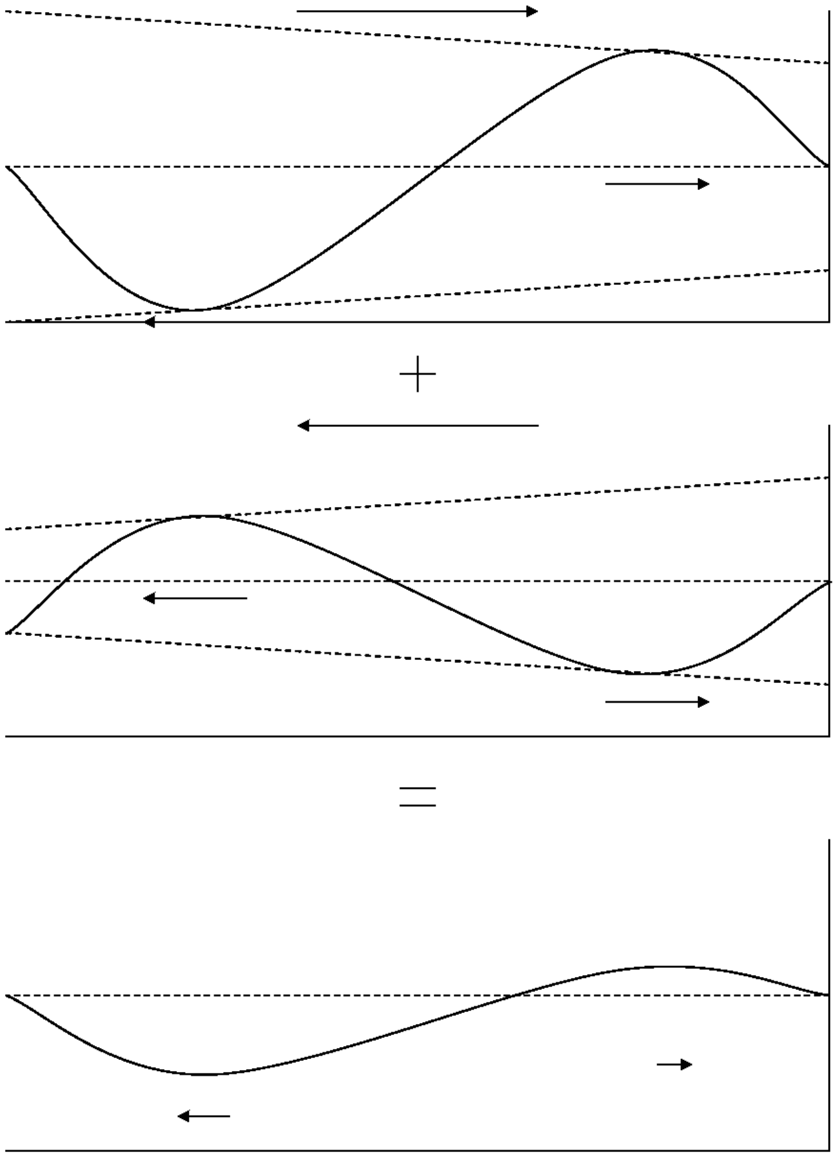

Figure 16. Damping of a tidal wave in an estuary

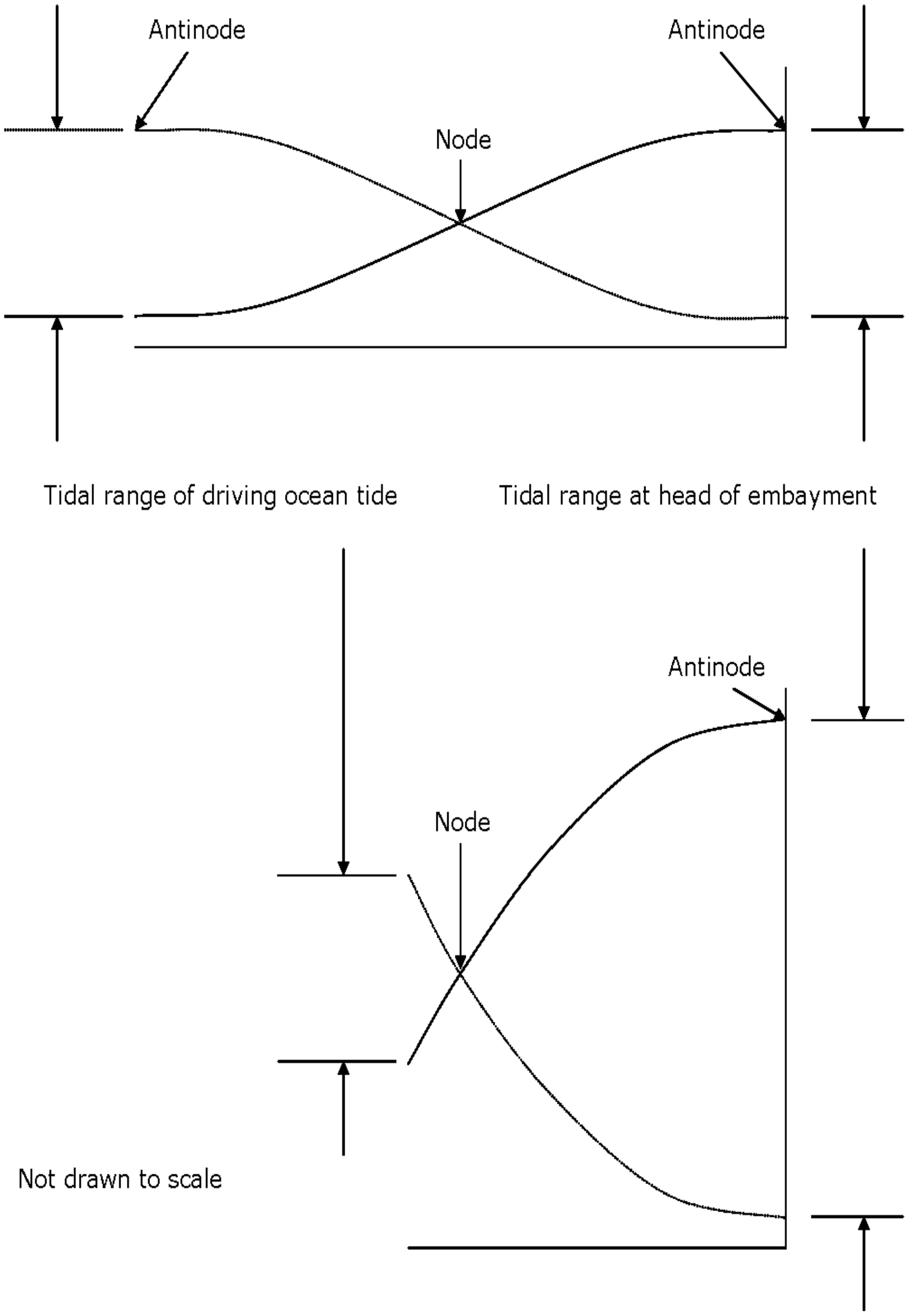

Figure 17. Relationship of tidal waves to size of estuaries

Tidal Bores

The tidal bore or eager (mascaret, French; pororoca, Brazilian) is a tidal wave that propagates up a relatively shallow and sloping estuary or river in a solitary wave form. The leading edge presents an abrupt rise in level, frequently with continuous breaking and often immediately followed by several undulations. An uncommon phenomenon, the tidal bore is usually associated with very large ranges in tide as well as wedge-shaped and rapidly shoaling entrances.

Bores occur in river estuaries with little or no reflections at the head. One of the models for a tidal bore is analogous to the process involved in an ordinary plunging breaker in the surf zone as described in Chapter 6. When a progressive tidal wave slows down to the point where the particulate motion (flood tidal current, u) in the crest (high tide) area exceeds the speed of the wave form itself, the tidal wave breaks. This is at a specific depth since $C = \sqrt{gh}$. There is a tremendous amount of water making up the high tide area that keeps coming in on the flood tidal current in contrast with the plunging breaker. As the tidal wave continues to propagate, the tidal height increases and, with a sloping riverbed, the point of breaking is just a little bit further up the estuary. This is a continuous process of infinitely small increments and it may continue for many miles up a river as a constantly breaking tidal bore (Figure 18).

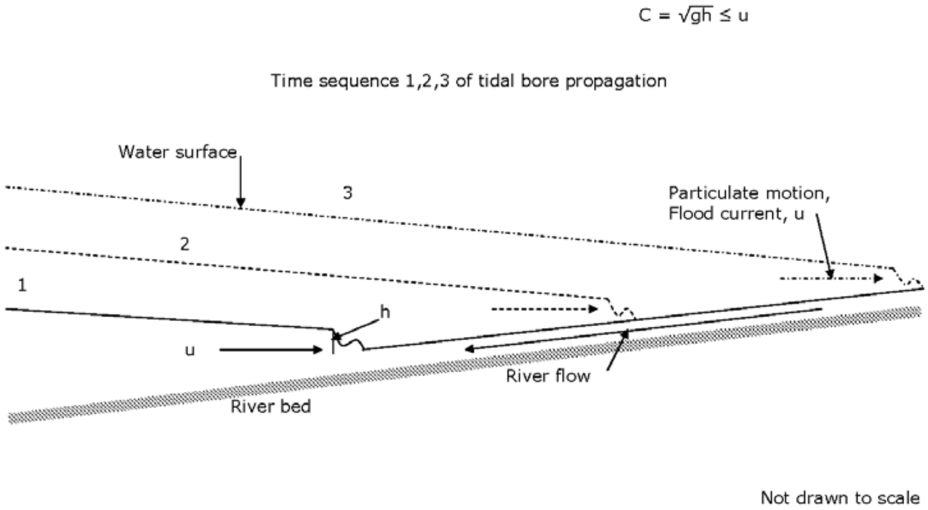

Figure 18. Propagation of a tidal bore in a river estuary

A very large tidal range is usually required. Bores tend to occur in river estuaries where funneling in the entrance topography helps to increase the range and the river flow tends to retard, and therefore build up, the wave. Also, tidal bores occur more often during spring tides and, especially, spring tides when the moon is in perigee.

Tidal bores occur in several river estuaries throughout the world. Before engineering modifications such as dredging, diversions, entrainment, and the construction of barriers and dams, many more existed. A few of the well known bores, together with the location in which they occur are:

Qiantang River, Haining Province, China
Petitcodiac, New Brunswick, Canada
Salmon River, Nova Scotia, Canada
Shubenacadie River, Nova Scotia, Canada
Solway Firth, between England and Scotland
Severn Estuary, England
River Trent, England
Orne River, France
Gironde River, France
Hooghly River, West Bengal, India
Amazon River, Brazil
Turnagain Arm, Cook Inlet, Alaska

CHAPTER 8

TIDAL HARMONIC CONSTITUENTS

Each of the tide-generating motions described in the above chapters can be represented by a simple cosine curve as illustrated in Figure 19. The horizontal axis represents time and the vertical represents the magnitude of the tide-generating force. The crests give the times of the maximums in the tide-generating force and the troughs, the minimums. For example, in the sun-earth system outlined in Chapter 3, noon, with the sun overhead, is at the first crest. Six hours later a minimum occurs at the trough. The second maximum is at midnight with the second crest. Another trough is for dawn and then back to the original noon crest. The vertical excursion of the curve is the range that the tide-generating forces are *trying* to cause in the waters from this component of the total tide.

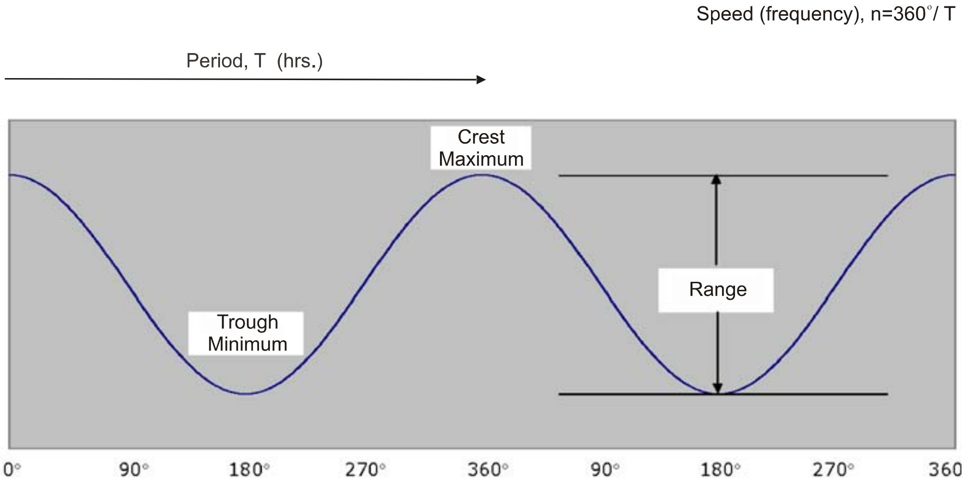

Figure 19. Constituent tide curve example

Each one of the tide-generating motions, represented by a simple harmonic cosine curve, is known as a tidal component, tidal constituent, or harmonic constituent. A letter or letters and usually a subscript is used to designate each constituent. The tidal constituent described above, for example, called the Principal Solar semidiurnal constituent, is designated S_2. The Principal Lunar semidiurnal constituent is designated M_2. S is for sun and M is for moon, of course, and the subscript $_2$ means that there are two complete tidal cycles for each astronomic cycle. Thus, these are said to be semidiurnal constituents. Constituents are described by their tidal period (the time from maximum to maximum), T. The period for the S_2 is 12.00 solar hours (hr.) and the period for the M_2 is 12.42 solar hours.

In tidal work, each constituent (cosine curve) is more often described by its speed (or frequency in degrees per hour). The cosine curve is divided into 360° (from crest to crest). The speed, n, of the constituent is 360°/T. Thus, for $S_{2,}$ $n = 360°/12.00 = 30°/hr$. For M_2, $n = 360°/12.4206 = 28.984°/hr$.

We could go on and on to describe almost all of the perturbations in the relative motions of the sun, moon, and earth (including the distance and declinational aspects described in Chapter 4). However, after about 37, the effects of these motions in representing the actual tides are extremely small in most locations. For tidally complex areas inside estuaries, such as Anchorage, AK, and Philadelphia, PA, it takes over one hundred constituents to adequately describe the tide curve. These additional constituents are artifacts that combine the fundamental diurnal and semidiurnal constituents to produce high frequency (from 3 to 13 cycles per day) constituents that attempt to describe the complex non-linear effects of bottom friction and shallow water.

Some of the prominent tidal harmonic constituents are now examined in considerable detail. Their representations of the various astronomical events and the development of their periods and speeds are essential in understanding the harmonic analysis techniques discussed the Chapter 9. The development of frictional, shallow water and compound tidal constituents are outside of the scope of this book. For more information about these and other constituents and a detailed discussion about harmonic analysis consult “Tidal Hydrodynamics” by Parker (1991).

As mentioned above, the Principal Solar semidiurnal constituent, S_2, represents the earth spinning relative to the sun. The earth rotates once in 24 mean solar hours or, since around the world is 360°, it is going at the rate of 360° / 24 = 15° / hr. However, there is a maximum in the solar tide producing force under the sun and again on the opposite side (midnight).

So, the period (maximum to maximum) of the constituent is 12 mean solar hours and the speed is:

S_2 360° / 12 = 30° / hr.

The Principal Lunar semidiurnal constituent, M_2, represents the earth spinning relative to the moon. Since the moon is moving eastward, it takes 24.8412 mean solar hours to bring the moon back overhead. Again, there are two maximums in this lunar day, so the period is only 12.4206 mean solar hours and its speed is:

M_2 360° / 12.4206 = 28.984° / hr.

S_2 and M_2 get into phase (maxima lined up) and out of phase (maximum of one lined up with the minimum of the other) to produce spring and neap tides, respectively (Figure 20). Spring tides occur at the times of full moon and new moon while neap tides occur at the times of the first and third quarter moons. The revolution of the moon around the earth relative to the sun takes 29.5306 days (called the synodic month or one lunation). Since there are two maximums, spring tides occur every 29.5306 / 2 = 14.765 days and neap tides 7.383 days later than the springs.

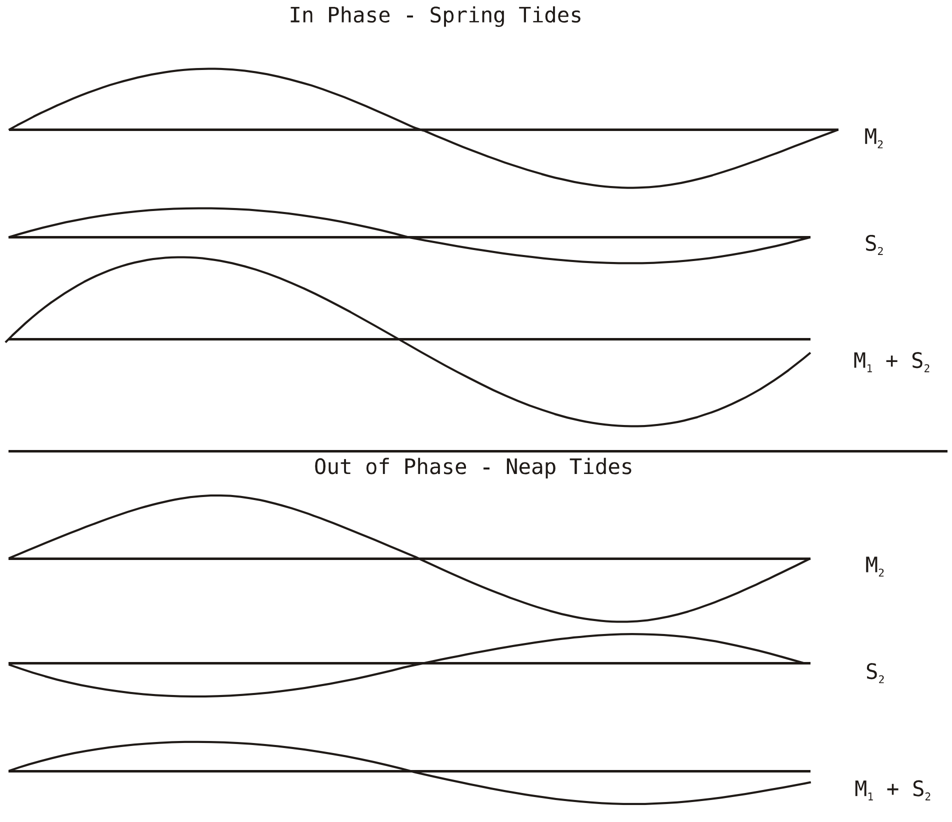

Figure 20. Phase relationships for spring and neap tides

The Larger Lunar Elliptic semidiurnal constituent, N_2, and the Smaller Lunar Elliptic semidiurnal constituent, L_2, are two constituents designed to simulate the cycle of perigee to perigee. These are completely artificial constituents in contrast with S_2 and M_2 that have realistic relationships to the solar and lunar envelopes of the tide-generating forces. Perigee to perigee occurs every 27.5546 days (the anomalistic month) or 661.31 mean solar hours. The speed of perigee to perigee is thus 360° / 661.31 = .544° / hr. This is a lunar event and the speed of M_2 is 28.984° / hr. The constituent speeds are, therefore:

N_2 28.984 – .544 = 28.440° / hr.

L_2 28.984 + .544 = 29.528° / hr.

Thus, when N_2 and L_2 are in phase every 27.5546 days (the anomalistic month) they add to M_2 to simulate the near approach to the moon (perigee). Also, 13.7773 days later they are out of phase simulating apogee (the moon farthest away).

The Luni-solar Declinational diurnal constituent, K_1, and the Principal Lunar Declinational diurnal constituent, O_1, are also artificial constituents designed to simulate the cycle of maximum declination to maximum declination of the moon. Maximum north to maximum north occurs every 27.3216 days (the tropical month) or 655.72 mean solar hours. However, both north and south declinations produce the same results. The north to south (and south to north) cycle is 655.72 / 2 = 327.86 hrs. The speed is 360° / 327.86 = 1.098° / hr. The speeds of the constituents, as they modify M_2, will be the speed of M_2 plus and minus the speed of the north to south cycle. Since the maximum is only felt once per day as the earth spins, the constituent speeds are half the sum and difference:

K_1 (28.984 + 1.098) / 2 = 15.041° / hr.

O_1 (28.984 – 1.098) / 2 = 13.943° / hr.

Thus, when K_1 and O_1 are in phase, every 13.6608 days (one half of the tropical month; i.e., the month with respect to the vernal equinox), they add to M_2 to simulate the maximum declination of the moon north or south. They account for the diurnal inequality due to the moon (the two high tides and/or the two low tides being unequal in height each tidal day) and, at extremes, diurnal tides (one high tide and one low tide each tidal day).

The Luni-solar Declinational diurnal constituent, K_1, and the Principal Solar Declinational diurnal constituent, P_1, are designed to simulate the cycle of maximum declination to maximum declination of the sun. Maximum north to maximum north occurs every 365.2422 days (the tropical year) or 8765.81 mean solar hours. However, both north and south declinations produce the same results. The north to south (and south to north) cycle is 8765.81 / 2 = 4382.91 hrs. The speed is 360° / 4382.91 = .082° / hr. The speeds of the constituents, as they modify S_2, will be the speed of S_2 plus and minus the speed of the north to south cycle. Since the maximum is only felt once per day as the earth spins, the constituent speeds are half of the sum and difference:

K_1 (30.000 + .082) / 2 = 15.041° / hr.

P_1 (30.000 – .082) / 2 = 14.959° / hr.

Thus, when K_1 and P_1 are in phase every 182.62 days (one half of the tropical year, i.e., the year with respect to the vernal equinox), they add to S_2 to simulate the maximum declination of the sun north or south. These constituents also contribute to diurnal inequality.

The speeds of the tidal harmonic constituents also may be derived by combining the speeds of certain fundamental astronomic elements. The classic description of the tide-producing forces uses a reference frame for which the earth is the center and projections of the movements of the sun and moon are made upon the celestial sphere. The fundamental astronomic elements are:

1. Mean rotation of earth relative to sun, T = 15° / mean solar hr.
2. Rate of change of moon, s = 0.549° / mean solar hr.
3. Rate of change of sun, h = 0.041° / mean solar hr.
4. Rate of change of lunar perigee, p = 0.005° / mean solar hr.

The speeds, n, of the same constituents described above are computed by this method:

Principal Solar semidiurnal constituent, S_2, n = 2T = 30° / hr.

Principal Lunar semidiurnal constituent, M_2, n = 2T – 2s + 2h = 28.984° / hr.

Larger Lunar Elliptic semidiurnal constituent, N_2, n = 2T – 3s + 2h + p = 28.440° / hr.

Smaller Lunar Elliptic semidiurnal constituent, L_2, n = 2T – s + 2h – p = 29.528° / hr.

Luni-solar Declinational diurnal constituent, K_1, n = T + h = 15.041° / hr.

Principal Lunar Declinational diurnal constituent, O_1, n = T – 2s + h = 13.943° / hr.

Principal Solar Declinational diurnal constituent, P_1, n = T – h = 14.959° / hr.

These are only two of several ways to develop tidal harmonic constituents and to compute their speeds. By going through each development, visualization and understanding is more easily obtained.

The theoretical relative magnitudes of the various constituents are also of interest. It must be remembered, however, that they are computed from the tide-generating forces and are not necessarily the values in the observed tide. They are based on the value, one, for M_2, since M_2 is usually the dominant constituent. The relative magnitude values, together with the periods of the constituents (360° / speed), are:

M_2	1.00	12.42 hrs.
S_2	0.46	12.00 hrs.
O_1	0.41	25.82 hrs.
K_1	0.40	23.93 hrs.
N_2	0.20	12.66 hrs.
P_1	0.19	24.07 hrs.
L_2	0.03	12.19 hrs.

The tidal harmonic constituents described in this chapter are only a few of many constituents. In practice, 37 constituents are used by CO-OPS for most locations to simulate the principal motions and perturbations in the sun-moon-earth system.

CHAPTER 9

HARMONIC ANALYSIS AND THE PREDICTION OF TIDES

Historical Organization

CO-OPS, and its predecessors, has always been the government agency responsible for water level and current observations, tide and tidal current analyses, tide and tidal current predictions, and sea level monitoring for the coastal waters of the United States and its possessions. The organization was known as Survey of the Coast from its founding in 1807 to 1836, Coast Survey to 1878, Coast and Geodetic Survey to 1970, and National Ocean Survey to 1982. In 1982 it was renamed National Ocean Service (NOS). From 1965 to 1970 the Coast and Geodetic Survey was a component of the Environmental Science Services Administration (ESSA). NOAA became the successor to ESSA in 1970.

Tide predictions by this organization began in 1844 using the lunitidal interval method. This method involved the time interval from the transit of the moon over the local meridian to the first high tide. Time adjustments were made for the phases of the moon in order to take into account some of the solar effects. In 1867 the harmonic method was introduced. The tedious additions of the cosine curves encouraged the invention of the mechanical Maxima and Minima Tide Predictor of William Ferrel in 1885. This machine summed 19 constituents. It is now in the National Museum of American History of the Smithsonian Institution. A second machine, developed by Rollin A. Harris and E. G. Fischer, summed 37 constituents and was used from 1912 through 1965. This one is now located in the NOS facilities in Silver Spring, MD, along with the first Tide Tables (1867) and other items of interest in the history of tides and tidal currents.

Harmonic Analysis

A water level (tide) station is a geographic location and facility at which water level heights are observed over time. Generally, the water levels are measured by a mechanical, acoustic, and/or electronic water level (tide) gauge of some sort. Peripheral equipment automatically records, stores, and/or transmits the data. The measurements are permanently referenced (vertically) to a series of bench marks on the adjacent land by terrestrial leveling.

Detailed information on these stations, their locations, the actual data, analyses of these data, tide and tidal current predictions, tidal datums, and general information on the tides are readily available at the CO-OPS web site, http://www.tidesandcurrents.noaa.gov/. Also included at this site is the Physical Oceanographic Real-Time System (PORTS®). PORTS® provides real-time water levels, currents, and certain meteorological and water (temperature and salinity) data over telephone and the Internet for harbor complexes that have PORTS® installed.

When water level data are plotted on a graph of height against time, a curve, somewhat resembling a cosine curve, is revealed. This curve contains all kinds of things. It includes the effects of currents, water density changes, and numerous meteorological factors as well as the hydrological effects from the proximity of rivers and estuaries.

Also included within this actual water level curve is the astronomic tide. To study it, the physical oceanographers of CO-OPS must remove the astronomic tide, usually the dominant signal, from the actual water level curve. They do it by taking out one constituent tide at a time. The process is a mathematical one called harmonic analysis. By knowing the periods of the constituents, it is possible to remove them, providing there is a series that is long enough. Generally, one year is desirable but one month can provide adequate results with dominant semidiurnal tides. Standard analyses are made for 37 constituents by CO-OPS, although several of them may be quite small at many of the stations.

From a harmonic analysis of the observed water level series, two values are obtained for each tidal constituent. Amplitude, the vertical distance between mean tide level and the level of the crest (when plotted as a cosine curve) is one of the values. The other value is the phase lag (Epoch). The phase lag is the amount of time elapsed from the maximum astronomic event to the first maximum of its corresponding constituent tide. It is usually expressed in degrees of one complete cosine curve (360°) of that constituent. These two values are known as harmonic constants and are illustrated in Figure 21. It must be remembered that they are unique to the particular station location from which they were derived. Also, the harmonic constants are treated as a constant even though in the strictest sense they are not because the computed values are affected by noise in the signal, the length of the series analyzed, etc. The accepted constants that are used are considered the best estimates of the actual (unknown) values. When any natural event or engineering project occurs, such as erosion, deposition, dredging, and breakwater construction that has the potential to cause major alterations in the adjacent topography, new measurements and a new harmonic analysis must be made. The

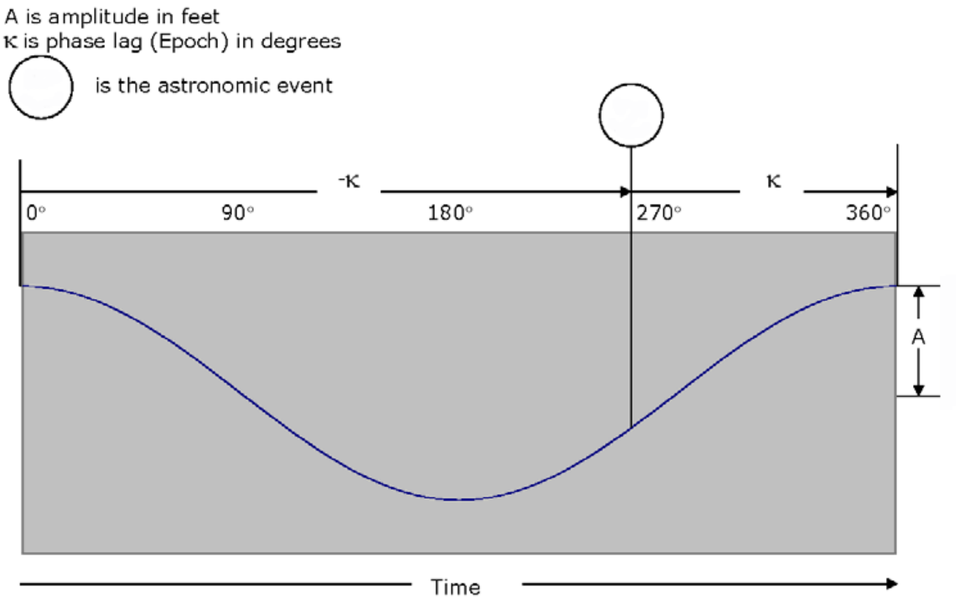

Figure 21. Phase lag and amplitude for a harmonic constituent

accepted harmonic constants that CO-OPS has for a number of station locations are listed in their web site under Observations – Other - Harmonic Constants.

Prediction of Tides

To predict the tide, say for a calendar year, it is necessary to know the harmonic constants (amplitudes and phase lags) for the constituents at each location for which predictions are desired. These are obtained from a harmonic analysis of the observed tide at each station as described above. Adjustments are made for the astronomic configurations for the beginning of the year. Knowing the phase lag of each constituent from the harmonic analysis, the first maximum of each cosine curve occurs after the event by the amount of its phase lag. The amplitude of each cosine curve is that found from the harmonic analysis. Finally, at each hour of the year, the heights of all the cosine curves are added. When plotted, the resulting curve is normally very similar (in shape and size) to the original observed curve.

The times and heights of the high tides and low tides are placed as predictions for the forthcoming year on the CO-OPS web site previously given. Predictions of the tide, called Water Level Tidal Predictions by CO-OPS, are given for over 3,000 locations. Tidal current predictions are given for over 2,700 locations. The vast number of predictions is possible by applying corrections to those stations for which harmonic constants have been determined - the Primary Control Stations (Reference Stations). Subordinate Stations (those without harmonic constants) are referred to their nearby Reference Stations by empirical constants. Thus, predictions are also obtained for these Subordinate Stations.

Mathematically, the process of prediction may be described as follows. To predict the tides at a particular location, it is simply a matter of adding the cosine curves of the tidal constituents. These constituents were obtained from the harmonic analysis of the tide, previously observed at that location. The harmonic analysis provides the tidal constants (amplitude of cosine curve and phase lag from the astronomic event) for each tidal constituent. This straightforward addition may be developed by:

$h = A \cos (at + \varphi)$ for one simple harmonic cosine curve where:
h is the height at any time t,
A is the amplitude of the curve,
a is the frequency (speed), and
φ is the phase at time 0.

For tides,
$h = H_0 + A_1 \cos (at + E - \kappa)_1 + A_2 \cos (at + E - \kappa)_2 + \cdots A_n \cos (at + E - \kappa)_n$

where:
H_0 is the height of mean water level above a selected datum,
$A_{1,2,n}$ is the amplitude of the constituent,
E is the equilibrium argument of a constituent at t = 0 (from the astronomic event to t = 0),
k is the phase lag or Epoch (from the astronomic event to the maximum amplitude of the constituent), and
n is the number of constituents used in the prediction - usually up to 37.

$E = V_0 + u = \text{Greenwich } (V_0 + u) + aS / 15 - pL$

where:
V_0 is the uniformly varying portion,
u is the slow variation due to changes in the longitude of the moon’s node,
S is west longitude of the time meridian,
L is west longitude of the station, and
p = 0,1,2,...n for long period, diurnal, semidiurnal, and other tides, respectively.

Types of Tide

Qualitatively, there are three basic types of tides. They are semidiurnal, mixed, and diurnal. When the two high tides and two low tides of each tidal day are approximately equal in height, the tide is semidiurnal. When there is a relatively large difference in the two high and/or low tides of each tidal day (i.e., large diurnal inequality), the tide is mixed. When there is only one high tide and one low tide each tidal day, the tide is diurnal.

The type of tide at a given location is largely a function of the declinations of the sun and moon. The declinations are constantly varying such that the type of tide changes throughout the month and year at many of the locations (see also Declination, Chapter 4).

A more rigorous classification system is available using the amplitudes of the major constituents at each location. Quantitatively, where the ratio of (K_1 + O_1) to (M_2 + S_2) is less than 0.25, the tide is classified as semidiurnal; where the ratio is from 0.25 to 1.5, the tide is mixed mainly semidiurnal; where the ratio is from 1.6 to 3.0, the tide is mixed mainly diurnal; and where the ratio is greater than 3.0, it is diurnal.

CHAPTER 10

TIDAL DATUMS

Tidal Datums

A datum is a reference from which linear measurements are made. It can be a physical point, line, or plane. It can also be an invisible point, line, or plane positioned by a statistical treatment of the numerical values of a particular natural phenomenon. Datums are usually specified as references for either horizontal or vertical measurements.

For marine applications, the sea surface is the vertical datum. It is used as the base elevation from which measurements of height and depth are made. However, since the sea surface moves up and down in time intervals of less than a tenth of a second to geological ages, and in height from less than an inch to many feet, this plane must be *statistically stopped* in its vertical excursions. When the sea surface is stopped by a statistical application to the numerical values of a particular phase (such as lower low water) of the tide, it is called a tidal datum.

As explained in previous chapters, there are usually two high waters (tides) and two low waters in each tidal day. The two high waters and/or two low waters are seldom equal in height, with the amount of the difference varying primarily with the moon's declination north and south of the equator over the month, and secondarily with the declination of the sun through the year. The difference in height between adjacent high and/or low waters is called diurnal inequality. The names of these phases are higher high water, lower high water, higher low water, and lower low water.

In some locations, the diurnal inequality becomes so large that the lower high and higher low waters actually disappear leaving only one tide per day, known as a diurnal tide. Diurnal tides are the least common. They usually occur in certain topographically susceptible regions and only during times of maximum declinational effects. Tides that are always diurnal are extremely rare.

The definition of a tidal datum, a method definition, generally specifies the mean of a particular phase or phases over a specific series length. For example, the tidal datum of mean lower low water is defined as the average of the lower low water heights (or only low water height) of each tidal day observed over the National Tidal Datum Epoch as defined by CO-OPS.

In order to make a datum stable, it is necessary to obtain an observed time series of sufficient length to average out the meteorological, oceanographic, and hydrologic variability as well as all significant tidal periods. A series length of 19 years was chosen because it contains all the tidal periods that need to be considered in tidal computations; i.e., through the 18.61-year node cycle (the period for the regression of the moon's nodes westward through 360° of longitude relative to the vernal equinox). Actually, the real problem was finding a rationale for selecting a series length that would effectively handle the meteorological, oceanographic, and hydrologic variability. A series length of 18.61 years seemed sufficient to handle the variability, and it did have an astronomic significance for the rationality. The series was then extended to 19 years because the amplitude of the annual cycle is huge in comparison with the amplitude of the node cycle. Also, the annual cycle is very short (very steep curve) in comparison with the 18.61-year node cycle (very flat curve). In other words, the change in height of the node cycle from 18.61 to 19 years is insignificant.

A specific 19-year period is necessary for standardization because of relative apparent secular trends in the sea level series. Relative means changes in the sea surface with respect to the adjacent land. Secular means non periodic and apparent secular means that it is not known whether the trend is truly secular or merely a segment of a much longer oscillation yet to be revealed. Also, a specific 19-year period is needed for standardization because relative apparent secular sea level trends are seldom linear; thus, if not used, meaningful comparisons between stations would be difficult. The specific 19-year period adopted by CO-OPS as the official time segment for tidal datum determinations is called the National Tidal Datum Epoch. It is currently 1983 through 2001. After 15 to 20 years or so, relative apparent secular trends in sea level cause tidal datum elevations to become unrealistic. Therefore, it is the policy of CO-OPS to update the tidal epoch every 25 years, or when needed to take into account the apparent secular (non-periodic) changes in sea level. Changes in sea level are monitored on a continuing basis and yearly changes are usually too small to have any practical significance.

Tidal datum is the accepted term for any one of a family of water surface elevation references. There are several tidal datums in common use today. The tidal datum of mean high water, for example, is the base elevation for structure heights, bridge clearances, etc. The tidal datum of mean lower low water has been officially designated chart datum for the United States, its territories, Commonwealth of Puerto Rico, and Trust Territory of the Pacific Islands. Chart datum is the datum for soundings and isobaths on all nautical charts and bathymetric maps, and for tide predictions.

Hydrographic Surveying

Hydrographic surveys are conducted to determine bottom topography. For each survey, numerous soundings are obtained, usually by ship or launch in a fairly small area along the coast and in a relatively short period of time. Horizontally, the soundings are controlled by precise positioning systems. Vertically, the soundings are controlled by tide measurements. The tide measurements are used in two parts. The first part is to obtain chart datum (mean lower low water). The second is to correct each sounding for the elevation of the water floating the ship above chart datum at the time each particular sounding is made. Thus, the corrected soundings will all be at depths of the water below chart datum.

It is not feasible to obtain a 19-year (let alone, specific 19-year) tide measurement series to determine chart datum in each segment of the coast for which hydrographic surveys are to be made. However, there are many tide stations distributed along the coasts that have been in operation for over 19 years. These stations are continuous and are considered permanent installations of CO-OPS. Each station is called a primary control tide station and collectively they comprise the present 117 marine stations of the National Water Level Observation Network (NWLON).

Essentially, each station consists of a tide gauge and a suite of tidal bench marks on the adjacent land. A large amount of ancillary equipment can also be present. The tidal bench marks are leveled between each other and to the gauge. Thus, the gauge measurements are all related to the relatively stable land nearby.

Secondary control tide stations (operated for at least one year) and tertiary tide stations (operated for at least one month) fill in the geographic gaps between primary control tide stations. Whether the gauge at a secondary or tertiary station is in operation or has been removed, its bench marks are retained in place. They are protected from destruction caused by vandalism, construction projects, and deterioration through respect for NOS (hopefully) and by the infrequent maintenance visits by CO-OPS tide personnel.

Even the secondary control and tertiary tide stations are probably not spaced at close enough intervals to be adjacent to many of the future hydrographic survey areas. Therefore, a method of comparison of simultaneous observations has been developed to extend the tidal datum system to new locations. *Note, this method is also used to compute accepted datums at secondary and tertiary stations.*

Whenever a primary control, secondary control, or tertiary tide station is not adjacent to an area in which a hydrographic survey is to be conducted, a temporary hydrographic tide station is established at a representative location for the area. The station is operated during the hydrographic survey. At the nearest control tide station, the vertical difference between the elevations of the tidal datum of mean tide level (the mean of all the high and low tides) over the 19-year National Tidal Datum Epoch (NTDE) and mean tide level over the same time period that the temporary hydrographic tide station was in operation, is computed. This difference is then applied to mean tide level at the hydrographic tide station to give the *equivalent* mean tide level over the NTDE at the hydrographic tide station location.

At the nearest control tide station, the mean diurnal range (mean difference between mean higher high water and mean lower low water) over the NTDE divided by the mean diurnal range over the same time period that the temporary hydrographic tide station was in operation, multiplied by the mean diurnal range at the hydrographic tide station, gives the *equivalent* mean diurnal range over the NTDE at the hydrographic tide station location. One-half of the equivalent mean diurnal range below the equivalent mean tide level is the elevation of mean lower low water – chart datum – at the hydrographic tide station.

The other half of the navigation equation is tide predictions. The nautical chart and bathymetric map provide the mariner with information on bottom topography over large areas through the use of printed discrete depth values and isobaths. These are *below* chart datum. Tide predictions provide the mariner with water level heights *above* chart datum as a function of date, time, and location. They are readily available on the CO-OPS website. With this depth information and tide predictions, the mariner can easily determine the distance from the water surface to the bottom for any time; both at his present location and at any other location he expects to go.

Coastal and Marine Boundaries

Coastal boundaries are formed by the intersection of the ocean surface with the land at the elevation of a particular tidal datum. To designate the boundary, the word “line” is used after the tidal datum name. For example, the boundary between private and state land, in most states, is the mean high water line. However, it should be remembered that the beach itself may erode or pro-grade due to such factors as wind waves, alongshore currents, storm surges, etc. Although this does not affect the tidal datum itself, it does affect the tidal datum line. In other words, in our example, the tidal datum of mean high water may remain constant, but the mean high water line (used as a coastal boundary) may move significant horizontal distances with erosion and depositional processes.

Marine (offshore) boundaries are also controlled by tidal datums. Basically, they are all measured from a tidal datum line. For example, the mean lower low water line forms the baseline from which most offshore boundaries are measured. Points on the mean lower low water line, as depicted on large scale nautical charts of NOS, are then used to strike distance arcs offshore to delineate the boundaries. These construction procedures follow predetermined national policy or international convention agreements. Figure 22 illustrates the many coastal and marine boundaries presently used by the United States.

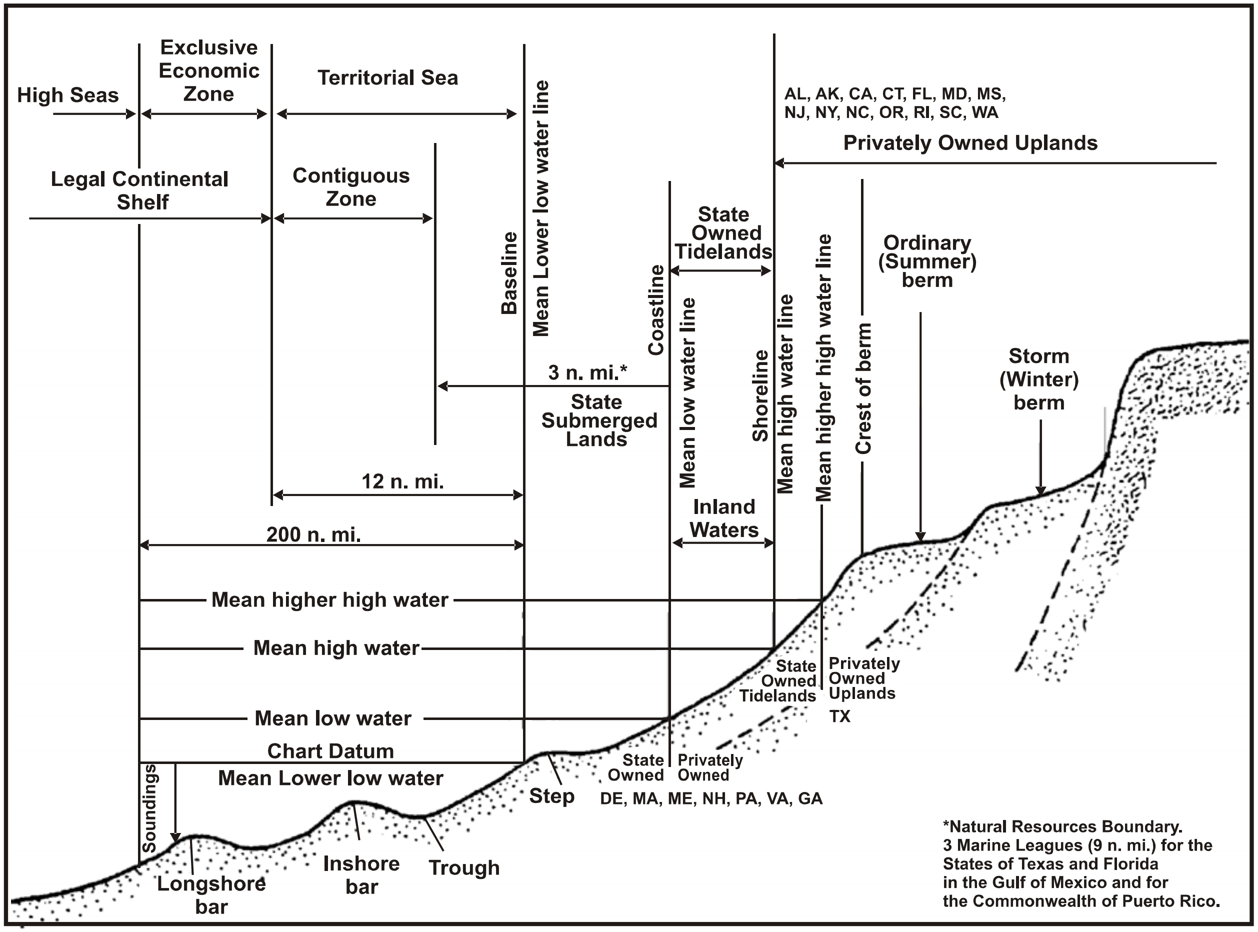

Figure 22. Coastal and marine boundaries from tidal datums.

Early Development of Tidal Datums and Marine Boundaries in the United States

President Thomas Jefferson appointed Ferdinand R. Hassler, a distinguished Swiss geodesist, the first Superintendent of the Survey of the Coast in 1807 on the recommendation of the American Philosophical Society. Essentially, the Survey was to delineate the shoreline and to conduct topographic and hydrographic surveys of the coast. The work did not begin until 1816 and consisted of baseline measurements and triangulation of greater New York Harbor. It lasted only to 1818. In 1832, the work began once more and by 1834 hydrographic surveys commenced. In all of his activities, Hassler used the most thorough, precise, and accurate procedures known in the world in his day. Similarly, he obtained the best instruments in existence anywhere and developed new ones when they did not meet his exacting standards.

The extreme precision required of Hassler for his associates and himself, was intrinsic to the determination of geographic positions (by astronomic observations), direct baseline measurements, triangulation, leveling, and tide measurements. With the geographic positions and key baseline measurements, triangulation could be conducted. The basic triangulation network would then be used for horizontal position control of the coastal topographic and hydrographic surveys.

A zero elevation was also needed for the topographic and hydrographic surveys. The surface of the ocean was the obvious answer but, with a tidal range of about 4 to 5 feet in the greater New York region, Hassler would have to mathematically stop the ocean surface in its vertical excursion in order to achieve his desired precision. This artificially stopped elevation of the ocean surface became what is now known as a tidal datum.

Hassler had another problem. The shoreline boundary that he was charged to delineate is formed by the intersection of the surface of the ocean with the land. Since the ocean surface went up and down with the tide, the shoreline moved up-the-beach-landward and down-the-beach-seaward. Furthermore, with a gently sloping beach, the horizontal excursion of the shoreline would be greatly magnified. The obvious solution was to stop the vertical motion with a tidal datum and thus, the horizontal motion would also be stopped.

Actually, our predecessors in England and along the east coast of North America were well aware of the tide and its importance to boundaries long before Hassler's day. They saw the rise and fall of the ocean amounting to several feet and the associated movement of the water's edge both inland and seaward amounting to many yards. Terms such as ordinary high tide line for the boundary between private and state lands (derived from English common law) were in general use. But it fell to Hassler and his successors down through the years to develop technically sound definitions for these vague legal terms. They would have to be *method definitions* and remain acceptable to the courts. These efforts slowly developed into our present tidal datum system. In fact, the system is still evolving. Increased economic pressure (population and offshore oil resources) on our coastal zone has stimulated ongoing research efforts to understand certain aspects of water level fluctuations and to continue incorporating the results of these investigations into the legal-technical tidal datum system.

Sea Level Variations

Sea level has a profound effect on coastal and marine boundaries. If changing sea level becomes great enough to cause CO-OPS to update the National Tidal Datum Epoch the datum elevations at some locations will change significantly. On a gently sloping beach, the horizontal displacement of the datum line will be greatly magnified. However, the boundaries do not change automatically with a new Epoch.

An increase in carbon dioxide accumulation in the upper atmosphere, primarily from the burning of fossil fuels, is a recent concern of the scientific community, Congress, and the general public. The concern is that the greenhouse effect will be intensified by this accumulation, thus slightly increasing the mean temperature of the earth. The result of this warming could include such things as shifts in climate belts with the concomitant adjustment of agricultural and fishery areas. Also predicted by model is an increase in the height of the surface of the ocean and the associated inundation of low lying coastal areas. The rise would be due to thermal expansion of the ocean waters and to the melting of glacial ice. The sensationalism of this predicted phenomenon tends to mask and to confuse a very important question; what has sea level *been* doing against the shores of the United States?

For very practical reasons, this question is extremely important. Since the rates of sea level rise have been fairly uniform, in most cases, short term projections are probably quite accurate. The size and/or amount of shore protection structures and beach nourishment required to mitigate that portion of erosion due to sea level rise is dependent on the local trend and variability of sea level. Private property lines, as well as state and federal coastal and marine boundaries, are dependent on changes in sea level elevation. Future requirements for drainage pumping and salt water intrusion prevention are a function of sea level trends. Hydrographic surveying is also controlled by the sea level stand. In conservation, our extremely biologically productive marshlands are threatened by the constant narrowing of the zone between the ocean and developed land due to rising sea level on the gently sloping shore. Finally, changes in trend will enable physical oceanographers to detect the beginning of the predicted greenhouse induced sea level rise.

Fortunately, CO-OPS keeps track of what sea level has been doing against the shores of the United States (Zervas, 2001). It operates 45 tide stations (from which sea level statistics are derived) that have been in continuous operation since at least 1940, 60 since at least 1950, and 122 since 1975. The longest sea level series is from San Francisco. It dates from 1854. The New York series began in 1856 but has only been continuous since 1893. The series at Seattle began in 1898. However, for station comparisons, the common 1950 to 1999 series is used.

The following abbreviated list gives the trend [mean rate of rise (or fall)] of sea level for various representative stations using the common 1950-1999 series (Zervas, 2001):

	mm/yr
Sewells Point (Norfolk), VA	4.48
Charleston, SC	3.05
Pensacola, FL	2.00
San Francisco, CA	2.23
Seattle, WA	2.26
Juneau, AK	–12.25
Honolulu, HI	1.44

The trends are what the ocean surface is actually doing on our beaches as observed by a person standing on a specific beach (relative sea level trends). They are the trends that affect our coasts and individual beaches and what coastal communities must react to.

Two of the largest components of relative sea level trends are: 1) local vertical land movements, and 2) changes in height of the sea surface relative to the geographic center of the earth. Models of the greenhouse effect predict an acceleration in the rise for the latter component. Therefore, one of the most important activities of CO-OPS is to study the changes in these sea level series each year. By careful analysis, it is hoped that the beginnings of the predicted acceleration (if it should occur) can be detected. If local vertical land movements do not have uniform accelerations themselves, then the ocean signal should be able to be separated out. However, with data through 1999, the predicted acceleration has not yet been detected.

CHRONOLOGY OF SIGNIFICANT TIDAL EVENTS IN THE UNITED STATES

When applicable, dates refer to the Tide or Tidal Current Table volume containing predictions for the stated year.

1807 The Survey of the Coast established.

1830 Tide predictions begin in the United States. One high tide time prediction per day for Boston, New York, and Charleston are published in The American Almanac. Time differences for 96 stations and spring ranges predictions for 84 stations are also given.

1836 The Survey of the Coast became Coast Survey.

1844 Tide Notes, including lunitidal intervals, appear on nautical charts of United States coasts and harbors.

1853 Tables for obtaining tide predictions by the lunitidal interval method are published in the Appendix of the Annual Report of the Superintendent of the U. S. Coast Survey for the first time.

1854 Tidal Division formed.

1864 Last year of publishing lunitidal interval tables. One thousand copies provided to Union Navy.

1867 First Tide Tables published by U. S. Coast Survey.

1868 Low tide predictions begin for west coast of Florida and Pacific coast.

1878 Coast Survey became Coast and Geodetic Survey.

1885 William Ferrel’s Maxima and Minima Tide Predictor introduced.

1887 Low tide predictions included for all stations in the Tide Tables.

1890 Tidal current predictions begin for New York harbor and vicinity.

1896 Tide Tables extended to include numerous ports throughout the world.

1910 Tidal Research Section formed.

1912 Harris-Fischer Tide Predicting Machine introduced.

1914 Last year Ferrel’s Maxima and Minima Tide Predictor used.

1922 Tide Table format changed from sequential listing to separate columns for high and low tides.

1923 Tidal Current Tables first published separately from Tide Tables. Published in two volumes – Atlantic Coast, North America, and Pacific Coast, North America.

1928 Single port miniature tables introduced.

1932 Last year of single port miniature tables (revived from 1940 through 1944 for New York harbor and vicinity only).

1940 Special restricted tables for war effort begin.

1951 Last year of special wartime occupation tables.

1955 Special tide tables for selected places in Greenland, Canada, and Alaska begin.

1958 Format changed from separate columns for high and low tides to sequential listing.

1959 Tide predictions added to Small Craft Chart series.

1961	Motor drive and automatic readout installed on Harris-Fischer machine. Last year of publication of special tide tables for selected places in Greenland, Canada, and Alaska.
1963	Established Deep Sea Tide Program.
1965	Last year Harris-Fischer Tide Predicting Machine used. Analog-to-digital recorder (ADR) tide gauges and computer processing introduced.
1966	Electronic digital computer introduced for predictions.
1967	International Symposium on Mean Sea Level of the IAPO and UNESCO held. Electronic digital computer introduced for harmonic analysis of tides.
1970	Coast and Geodetic Survey became National Ocean Survey.
1973	Sea Level Service begins. Established National Tidal Datum Epoch.
1977	Gulf Coast Low Water Datum adopted.
1978	Water Level Telemetry System introduced for marine coasts.
1980	National Tidal Datum Convention of 1980 adopted.
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1982	National Ocean Survey became National Ocean Service. Personal computer software introduced for local user access to water level telemetry stations.
1988	Personal computer software introduced for local user access to current telemetry stations. Operational Next Generation Water Level Measurement System (NGWLMS) field units introduced using air acoustic ranging sensor and satellite telemetry.

1995 Last year of Tide Table and Tidal Current Table publication in hard copy by CO-OPS, NOS, NOAA. Partial tide prediction provided on telephone bulletin board.

1996 CO-OPS tide and tidal current predictions begun exclusively on Internet and CDs. Private publishers provided CO-OPS predictions for various tide and tidal current publications. CO-OPS tide and tidal current predictions provided to National Imagery and Mapping Agency for publishing in hard copy for U.S. Navy, U.S. Coast Guard, etc.

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